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## Analysis of MHD Free Convection Flow within Coaxial Cylinders with Nonlinear Thermal Radiation and Porous Material Effects

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### Abstract

An investigation of an electrically conducting viscous fluid within an annulus incorporating the influence of a radial magnetic field using the Rosseland diffusion approximation with porous material has been carried out. Both steady and unsteady solutions are obtained by applying regular perturbation and implicit finite difference methods. The key findings of this research shows that suction decreases temperature and velocity, while injection increases them. Magnetic fields reduce velocity, whereas higher Darcy number, Grashof, and radiation parameters enhances it especially under isothermal conditions.

**Keywords:** Porous Material, Annulus, Coaxial cylinder.

### 1. Introduction

The study of Magnetohydrodynamics (MHD) free convective flow within coaxial cylinders is of significant interest in both engineering and scientific applications. This investigation focuses on the influence of nonlinear thermal radiation with porous material focusing the behavior of the flow under isothermal and isoflux boundary conditions. Understanding the interaction between magnetic fields and electrically conducting fluids in this context is crucial for applications ranging from industrial processes to astrophysical phenomena. The complex relationship between thermal radiation and convective heat transfer within the coaxial cylinder configuration presents a rich area for exploration, especially when considering the effects of nonlinear Rosseland diffusion approximation.

Nonlinear Rosseland diffusion approximation is more accurate in situations where the opacity of the medium varies significantly with temperature, this approximation allows more complex and variable relationships between parameters, making it more suitable for cases with large temperature variations and highly temperature sensitive opacities. Several authors explored on the effect of nonlinear Rosseland diffusion approximation, Isah et al. (2018) studied Couette flow under the influence of the transverse magnetic field and thermal radiation for a nonlinear coupled system of two infinite vertical plates held at different temperatures. Abdullahi et al. (2022) studied the effect of Transient MHD heat transfer within radiative channel due to

convective boundary and slip velocity with non-linear Roseland approximation to describe the radiative heat flux in the energy equation. Their finding indicates that the convective boundary condition and slip velocity has a great impact on the flow parameters. Isah et al. (2023) investigated on the upshot of nonlinear thermal radiation of steady state MHD heat transfer of Jeffrey nanofluid together with prescribed boundary conditions. They stated that the thermal radiation parameter and heat source-sink parameter improves the flow temperature. Recent studies identifying this research field can be traced in References (Abu zeid et al. (2019), Govind and Rakesh (2020), Kaladhar et al. (2020), Isah et al. (2022), Isah et al. (2023), Michael et al. (2023), Isah et al. (2024), and Isah et al. (2025)).

The influence of MHD free convective flow has been systematically examined by many researchers, Gireesha and Sindhu (2020) reported on the MHD natural convection flow of Casson fluid in an annular Microchannel containing porous medium with heat generation/absorption, they report that higher values of Casson parameter increases the skin friction coefficient it is also observed that the rate of heat transfer reduces as fluid wall interaction parameter upsurges. The effect of MHD free convective flow can be presented in references (Jha et al., 2011), Jha et al. (2012), Gambo and Gambo (2020), Gao et al. (2020), and Abubakar et al. (2025).

The aim of this research is to study the analysis of thermal Rosseland diffusion approximation, porous material at the outer surface of the inner cylinder and the inner surface of the outer cylinder, an explanation on how different relevant flow parameters significantly affect the rate of flow processes is provided and shown using figures and tables. This will contribute to the broader understanding of thermal management and heat transfer in systems where porous material and thermal radiation play a critical role.

## 2. Mathematical Model

The physical problem under consideration consists of a transient natural convection flow in an infinite vertical annulus formed by two infinite permeable vertical cylinders in presence of thermal radiation under a radial magnetic field. The transient flow formation of outer surface of inner cylinder is suddenly heated isothermally or at a constant heat flux. The physical properties of the working fluid are assumed to be constant. At  $t' \leq 0$ , both the fluid cylinder are assumed to be at the same temperature  $T_0$  with absence of fluid motion. At time  $t' > 0$ , the temperature of the outer surface of inner cylinder at  $r = a$  is sudden raised to  $T_w$  ( $T_w > T_0$ ) causing transient free convection current. The  $z$  - axis, which is vertically upward, parallel to the gravitational attraction is taken as the axis of the cylinder, whereas the  $r'$  -axis is taken in the radial direction. Since the flow is fully developed and the cylinders are of infinite length, the flow depends on radial coordinate ( $r'$ ) and ( $t'$ ), see Figure 1. In line with the Boussinesqs approximation, the governing equations in dimensional form are:

$$\frac{\partial u'}{\partial t'} + \frac{V_0}{r'} \frac{\partial u'}{\partial r'} = \nu \left[ \frac{1}{r'} \frac{\partial}{\partial r'} \left( r' \frac{\partial u'}{\partial r'} \right) \right] - \frac{1}{r'^2} \left( \frac{\nu}{K^* a'} + \mu_e B_0 a' \sqrt{\frac{\sigma}{\mu}} \right) u' + g \beta (T' - T_\infty). \quad (1)$$

$$\frac{\partial T'}{\partial t'} + \frac{V_0}{r'} \frac{\partial T'}{\partial r'} = \alpha \left[ \frac{1}{r'} \frac{\partial}{\partial r'} \left( r' \frac{\partial T'}{\partial r'} \right) - \frac{1}{K} \frac{\partial q_r}{\partial r'} \right]. \quad (2)$$

The radiation heat flux term  $q_r$  in the righthand side of equation (2) is simplified using the Roseland diffusion approximation:

$$q_r = \frac{4\sigma\partial T'^4}{3k^*\partial y'} \quad (3)$$

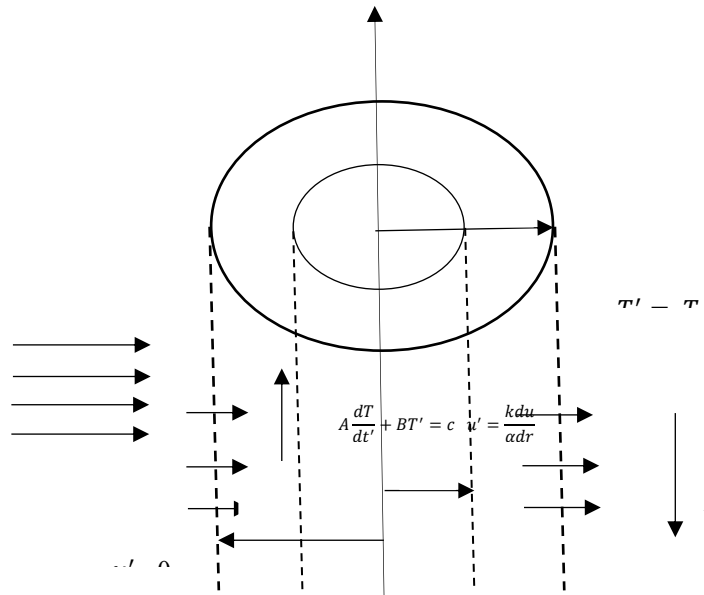


Figure 1: Schematic diagram for problem under consideration

The dimensional initial and boundary conditions for the present problem assumed the following form:

$$t' < 0: u' = 0, T' = T_0, \text{ for } a^* \leq r' \leq b^* \\ t' > 0: \begin{cases} u' = \frac{k}{\alpha} \frac{du'}{dr'}, \frac{dT'}{dr'} = -\frac{q'}{k}, \text{ or } T' = T_w \text{ at } r' = a^* \\ u' = 0, T' = T_0 \text{ at } r' = b^* \end{cases} \quad (4)$$

To obtain the solutions of equations (1) and (2) in dimensionless form subject to the boundary conditions in equation (4), the following dimensionless variables are presented to the problem:

$$u = \frac{u'}{v}, \quad r = \frac{r'}{a}, \quad \lambda = \frac{b}{a}, \quad t = \frac{t'v}{a^2}, \quad \text{Re} = \frac{V_0 a}{v}, \quad \theta = \frac{T' - T_\infty}{\Delta T}, \quad R = \frac{4\sigma_{sb}(\Delta T)^3}{k^* K}, \\ C_T = \frac{T_0}{T_w - T_0}, \quad M^2 = \frac{\mu B_0 a}{v} \sqrt{\frac{\sigma}{\mu}}, \quad \eta = \frac{k}{\alpha a}, \quad Gr = \frac{g\beta(T' - T_\infty)a^3}{v^2}, \quad v = \frac{\mu L}{\rho}, \quad Da = \frac{K^* a'}{v} \quad (5)$$

Where  $\Delta T = T_w - T_0$  or  $\frac{aq'}{k}$  consequently, as the inner cylinder is maintained at a constant temperature

$T_w$  or constant heat flux  $q_0$  respectively. Applying the dimensionless quantities introduced in equation (5) into equation (1) and equation (2) the following new mathematical models were derived:

$$\frac{\partial u}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) - \frac{\text{Re}}{r} \frac{\partial u}{\partial r} - \frac{1}{r^2} \left( \frac{1}{\text{Da}} + M^2 \right) u + \text{Gr} \theta. \quad (6)$$

$$\text{Pr} \frac{\partial \theta}{\partial t} = \left[ 1 + \frac{4}{3} R (C_T + \theta)^3 \right] \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) \right) + 4R (C_T + \theta)^2 \left( \frac{\partial \theta}{\partial r} \right)^2 - \frac{\text{Re}}{r} \frac{\partial \theta}{\partial r}. \quad (7)$$

Subject to the following dimensionless boundary conditions

$$\left. \begin{aligned} u &= \eta \frac{\partial u}{\partial r}, \quad A \frac{\partial \theta}{\partial r} + B\theta = C \quad \text{at } r = 1 \\ u &= 0, \quad \theta = 0 \quad \text{at } r = \lambda \end{aligned} \right\}. \quad (8)$$

The model equations deposited in equations (6) and (7) are coupled and highly nonlinear which cannot be solved in closed form. Therefore, for the purpose of this study two method will be adopted namely:

### 3. Analytical Solution

In order to represent the steady-state version of equations (6) and (7) with suitable boundary conditions as  $t \rightarrow 0$ , the MHD-free convective flow within the coaxial cylinder due to nonlinear thermal radiation in an isothermal/isoflux condition can be treated as follows:

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{du}{dr} \right) - \frac{\text{Re}}{r} \frac{du}{dr} - \frac{1}{r^2} \left( \frac{1}{\text{Da}} + M^2 \right) u + \text{Gr} \theta = 0. \quad (9)$$

$$\left[ 1 + \frac{4}{3} R (C_T + \theta)^3 \right] \left( \frac{1}{r} \frac{d}{dr} \left( r \frac{d\theta}{dr} \right) \right) + 4R (C_T + \theta)^2 \left( \frac{d\theta}{dr} \right)^2 - \frac{\text{Re}}{r} \frac{d\theta}{dr} = 0. \quad (10)$$

Subject to the following boundary conditions

$$\left. \begin{aligned} u &= \eta \frac{du}{dr}, \quad A \frac{d\theta}{dr} + B\theta = C \quad \text{at } r = 1 \\ u &= 0, \quad \theta = 0 \quad \text{at } r = \lambda \end{aligned} \right\}. \quad (11)$$

The assumed solutions of equations (9) and (10) analytically bear perturbation method such that a negligible thermal radiation ( $R \ll 1$ ) parameter is use subject to the boundary condition in equation (11) can be written as :

$$\left. \begin{aligned} u(r) &= u_0 + \sum_{i=1}^{\infty} R^i u_i(r) \\ \theta(r) &= \theta_0 + \sum_{i=1}^{\infty} R^i \theta_i(r) \end{aligned} \right\} \quad (12)$$

Substituting equations (9) and (10) into equations (12) and neglecting the highest order of powers of  $R$  will give the required analytical expressions for both velocity and temperature equations subject to the boundary condition in equation (11) respectively as follows:

$$u(r) = s_3 r^{s_1} + s_4 r^{s_2} + s_5 r^2 + s_6 r^{2+Re} + R(s_{13} r^{s_1} + s_{14} r^{s_2} + s_{15} \ln r r^{2+Re} + s_{16} r^{2+2Re} + s_{17} r^{2+3Re} + s_{18} r^{2+4Re} + s_{19} r^2 + s_{20} r^{2+Re}) \quad (13)$$

$$\theta(r) = h_3 + h_4 r^{Re} + R(h_{11} \ln r r^{Re} + h_{12} r^{2Re} + h_{13} r^{3Re} + h_{14} r^{4Re} - h_9^{-Re} + h_{10} r^{Re}). \quad (14)$$

The drag coefficients for the steady state at the outer surface of the inner cylinder and the inner surface of the outer cylinder are derived from equation (13) as follows:

$$\tau_1 = \frac{du}{dr} \Big|_{r=1} = s_3 s_1 + s_4 s_2 + 2s_5 + (2 + Re)s_6 + R[s_{13} s_1 + s_{14} s_2 + s_{15} + (2 + 2Re)s_{16} + (2 + 3Re)s_{17} + (2 + 4Re)s_{18} + 2s_{19} + (2 + Re)s_{20}] \quad (15)$$

$$\tau_\lambda = \frac{du}{dr} \Big|_{r=\lambda} = s_3 s_1 \lambda^{s_1-1} + s_4 s_2 \lambda^{s_2-1} + 2s_5 \lambda + (2 + Re)s_6 \lambda^{1+Re} + R[s_{13} s_1 \lambda^{s_1-1} + s_{14} s_2 \lambda^{s_2-1} + s_{15} [(2 + Re) \ln \lambda \lambda^{1+Re} + \lambda^{2+Re}] + (2 + 2Re)s_{16} \lambda^{1+2Re} + (2 + 3Re)s_{17} \lambda^{1+3Re} + (2 + 4Re)s_{18} \lambda^{1+4Re} + 2s_{19} \lambda + (2 + Re)s_{20} \lambda^{1+Re}]. \quad (16)$$

Likewise, the rate of heat transfer for the steady state at the outer surface of the inner cylinder and inner surface of the outer cylinder are derived from equation (14) as follows:

$$Nu_1 = \frac{d\theta}{dr} \Big|_{r=1} = Re h_4 + R(h_5 + h_{15} + h_{16} + h_{17} + Re h_{10}). \quad (17)$$

$$Nu_\lambda = \frac{d\theta}{dr} \Big|_{r=\lambda} = Re h_4 \lambda^{Re-1} + R[h_5 (\ln \lambda \lambda^{Re-1} + \lambda^{-\lambda^{Re}}) + h_{15} \lambda^{2Re-1} + h_{16} \lambda^{3Re-1} + h_{17} \lambda^{4Re-1} + Re h_{10} \lambda^{Re-1}]. \quad (18)$$

#### 4. Numerical Solution

The time derivatives and second-order space derivatives of the momentum and energy equations in equations (6) and (7) can be visualized as follows:

$$\frac{u_i^{j+1} - u_i^j}{\Delta t} = \frac{u_{i-1}^{j+1} - 2u_i^{j+1} + u_{i+1}^{j+1}}{(\Delta r)^2} + \frac{k}{r} \left( \frac{u_{i+1}^j - u_{i-1}^j}{2\Delta r} \right) - \left( \frac{1}{Da} + M^2 \right) \frac{1}{r^2} u_i^j + Gr \theta_i^j. \quad (19)$$

$$\Pr\left(\frac{\theta_i^{j+1} - \theta_i^j}{\Delta t}\right) = z_1 \left( \frac{\theta_{i-1}^{j+1} - 2\theta_i^{j+1} + \theta_{i+1}^{j+1}}{(\Delta r)^2} \right) + \frac{z_1}{r} \left( \frac{\theta_{i+1}^j - \theta_{i-1}^j}{2\Delta r} \right) + 4R(C_T + \theta_i^j)^2 \frac{(\theta_{i+1}^j - \theta_{i-1}^j)^2}{4\Delta r^2} - \frac{Re^*}{r} \left( \frac{\theta_{i+1}^j - \theta_{i-1}^j}{2\Delta r} \right). \quad (20)$$

Subject to the following boundary conditions:

$$\left. \begin{aligned} u_i^{j+1} &= \eta \left( \frac{-3u_{i-1}^{j+1} + 4u_i^{j+1} - u_{i+1}^{j+1}}{2\Delta r} \right), \quad A \left( \frac{-3\theta_{i-1}^{j+1} + 4\theta_i^{j+1} - \theta_{i+1}^{j+1}}{2\Delta r} \right) + B\theta_i^{j+1} = C \quad \text{at } j=1 \\ u_i^{j+1} &= 0, \quad \theta_i^{j+1} = 0 \quad \text{at } j=\lambda \end{aligned} \right\}. \quad (21)$$

Further simplification of equations (19) and (20) gives rise to the following equations:

$$V_l u_{i-1}^{j+1} + V_c u_i^{j+1} V_r u_{i+1}^{j+1} = \left( 1 - r_3 \left( \frac{1}{Da} + M^2 \right) \right) u_i^j + r_2 k u_{i+1}^j - r_2 k u_{i-1}^j + \Delta t Gr \theta_i^j. \quad (22)$$

$$\begin{aligned} T_l \theta_{i-1}^{j+1} + T_c \theta_i^{j+1} + T_r \theta_{i+1}^{j+1} &= \Pr \theta_i^j + z_1 r_2 (\theta_{i+1}^j - \theta_{i-1}^j) \\ &+ Rr_1 (C_T + \theta_i^j)^2 (\theta_{i+1}^j - \theta_{i-1}^j)^2 - Re r_2 (\theta_{i+1}^j - \theta_{i-1}^j). \end{aligned} \quad (23)$$

Transforming equations (22) and (23) using equation (21) when  $i=1$  yields:

$$\begin{aligned} \left( \frac{4}{3} V_l - \frac{2\Delta r}{3\eta} V_l + V_c \right) u_i^{j+1} + \left( V_r - \frac{V_l}{3} \right) u_{i+1}^{j+1} &= \\ \left( 1 - r_3 \left( \frac{1}{Da} + M^2 \right) - \frac{4}{3} r_2 k + \frac{2\Delta r r_2 k}{3\eta} \right) u_i^j + \left( r_2 k + r_2 \frac{k}{3} \right) u_{i+1}^j + \Delta t Gr \theta_i^j. \end{aligned} \quad (24)$$

$$\begin{aligned} [z_0 T_l + T_c] \theta_i^{j+1} + \left( T_r - \frac{T_l}{3} \right) \theta_{i+1}^{j+1} - \frac{2\Delta r C}{3A} T_l &= \\ z_2 \theta_i^j + z_3 \theta_{i+1}^j + z_4 + Rr_1 (C_T + \theta_i^j)^2 \left[ \theta_{i+1}^j - z_0 \theta_i^j + \frac{1}{3} \theta_{i+1}^j + 2 \frac{\Delta r C}{3A} \right]^2. \end{aligned} \quad (25)$$

The scheme is convergent and unconditionally stable, without imposing any restrictions on the mesh ratio. Given an initial temperature estimate at the grid points, the system reduces to a tridiagonal matrix. The temperature field is then determined using the identified values from the previous time step  $t_i$  for all  $i=1, 2, \dots, M$ . The obtained temperature field is subsequently used to calculate the velocity. This process is repeated until the temperature and velocity converge to the desired solution (Isah et al., 2022).

## 5. Result and Discussion

The analysis of the MHD-free convective flow with thermal Rosseland diffusion approximation and porous material in an isothermal and isoflux condition within a coaxial cylinder has been conducted both analytically, utilizing the perturbation technique, and numerically, using the finite difference method. The thermal Rosseland diffusion approximation was employed to represent the radiative heat flux, taking into account the temperature difference parameter  $C_T$ . Validation of the numerical scheme was performed using the analytical result, demonstrating excellent agreement between the numerical and analytical solutions with a negligible thermal radiation parameter. In this study, parametric values were considered over a reasonable range:  $-3.0 \leq Re^* \leq 3.0$ ,  $Re^* > 0$  is indicating injection and  $Re^* < 0$  is indicating suction,  $2.0 \leq M \leq 4.0$ , the radiation parameter  $R$  falls within the range of  $0 \leq R \leq 0.8$ , Since the terms related to  $R$  act as intense heat sources, a high value of  $R$  can cause the temperature to rapidly increase and reach a finite-time blow-up see (Makinde and Chinyoka (2010) and Jha et al. (2018)). However time is taken within the range of  $0.2 \leq t \leq 2.0$ , temperature difference parameter  $0.01 \leq C_T \leq 1.5$ , and prandtl number  $Pr = 0.71$ ,  $2.0 \leq Gr \leq 4.0$ ,  $Gr = 0$  occurs when the flow is generated by movement from one boundary,  $Gr < 0$  corresponds to the external heating of the moving plate while  $Gr > 0$  corresponds to the external cooling of the moving plate, slip parameter carried the range of  $0 \leq \eta \leq 0.9$ , Darcy number falls within the range of  $0.01 \leq Da \leq 0.08$ , with the following default values:  $R = 0.01$ ,  $t = 0.02$ ,  $Re^* = -2.0$ , or  $Re^* = 2.0$ ,  $M = 2.0$ ,  $Gr = 2.0$ ,  $\eta = 0.5$ ,  $C_T = 0.01$ .

## 6. Velocity profiles

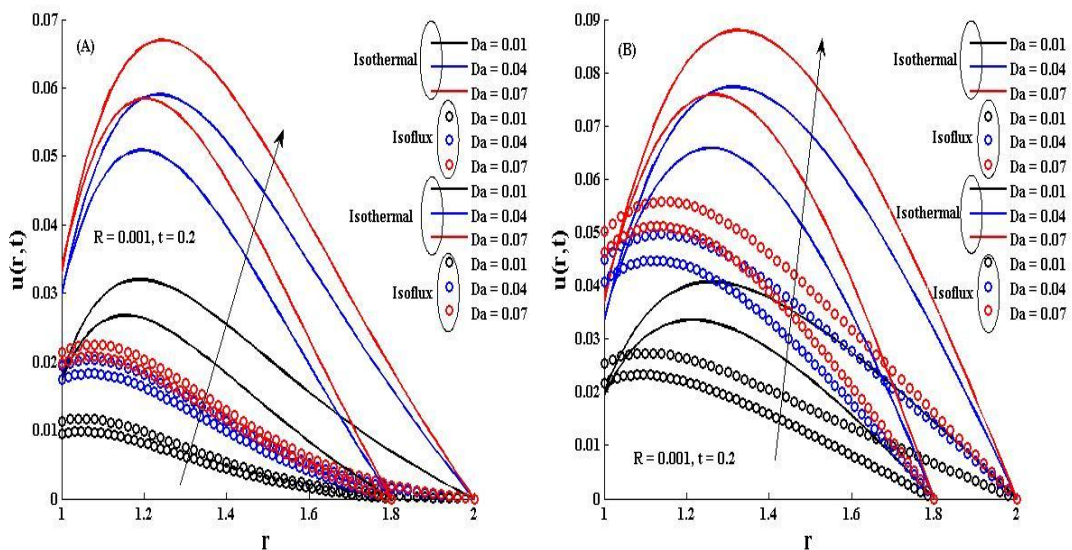


Figure 2 presents a side view of velocity, highlighting the variation in values for Darcy parameter, suction (A), and injection (B)

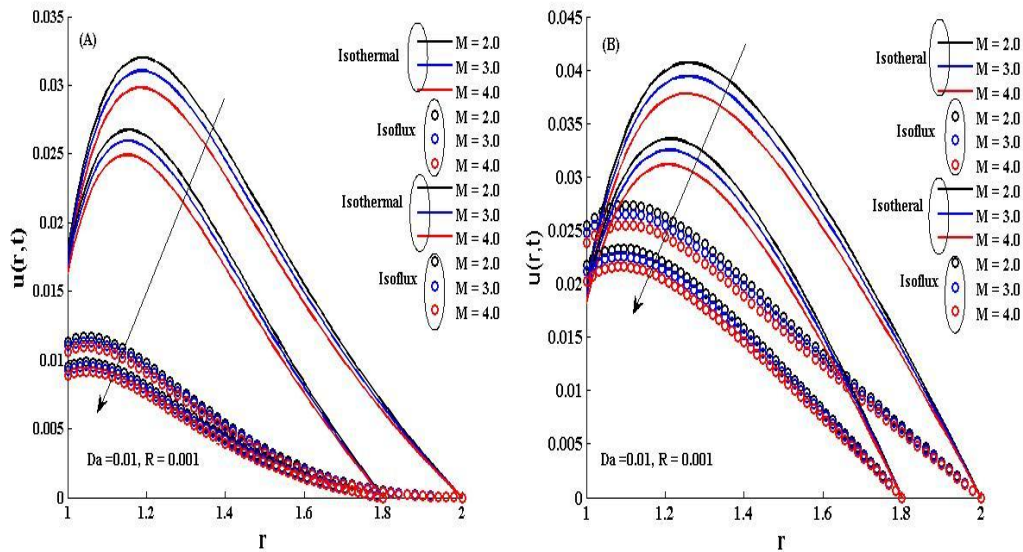


Figure 3 presents a side view of velocity, highlighting the variation in values for Magnetic parameter, suction (A), and injection (B)

Figure 2 illustrates the effect of Darcy parameter on the velocity profile, 2 (A) suction and 2(B) injection. It is noticeable in Figure 2(A) and 2(B) that increase in the Darcy parameter results in more rapid increase in velocity near the boundary, this is because the increased permeability and decreased viscosity allow the fluid to flow more easily through the porous medium, thus, the effect is perceptible in Figure 2(B), in the presence of injection.

Figure 3 presents the velocity profiles for variations in the magnetic parameter, Figure 3(A) shows suction, and Figure 3(B) shows injection, both under isothermal and isoflux conditions. It was observed that an increase in the magnetic parameter in Figure 3(A) results in a decrease in the velocity profile, particularly under isoflux conditions. A similar trend is observed in Figure 3(B), where increasing the magnetic parameter reduces the velocity profile despite the presence of injection. This reduction is due to the resistive force known as the Lorentz force, which appears in the momentum equation.

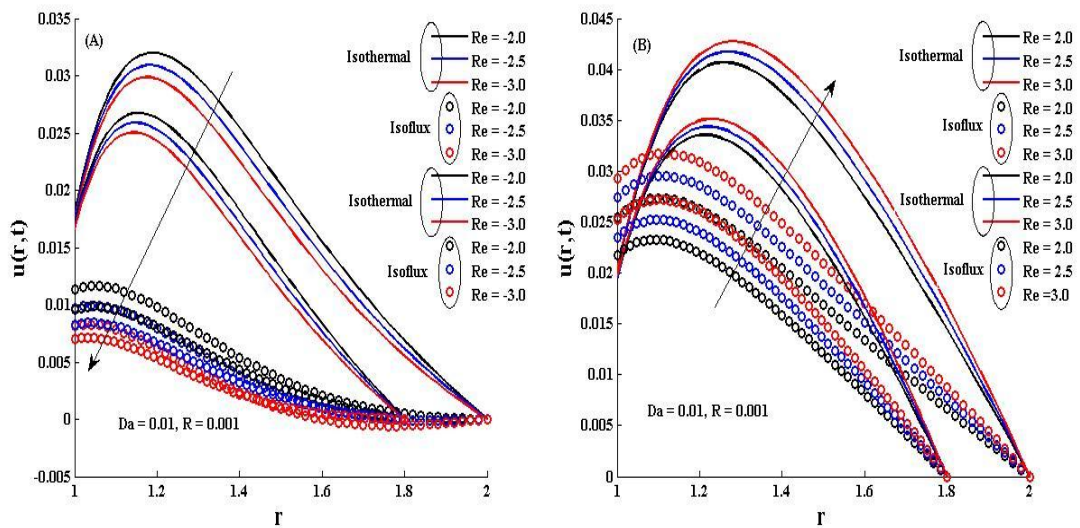


Figure 4 presents a side view of velocity, highlighting the variation in values for suction (A) and injection (B)

Figure 4 shows the effects of suction Figure 4(A) and injection Figure 4(B) on the velocity profile. In Figure 4(A), increasing the suction parameter results in a reduction in the velocity profile under both isothermal and isoflux conditions. This reduction occurs due to the thinning the boundary layer, leading to steeper velocity gradients, which affects the velocity.

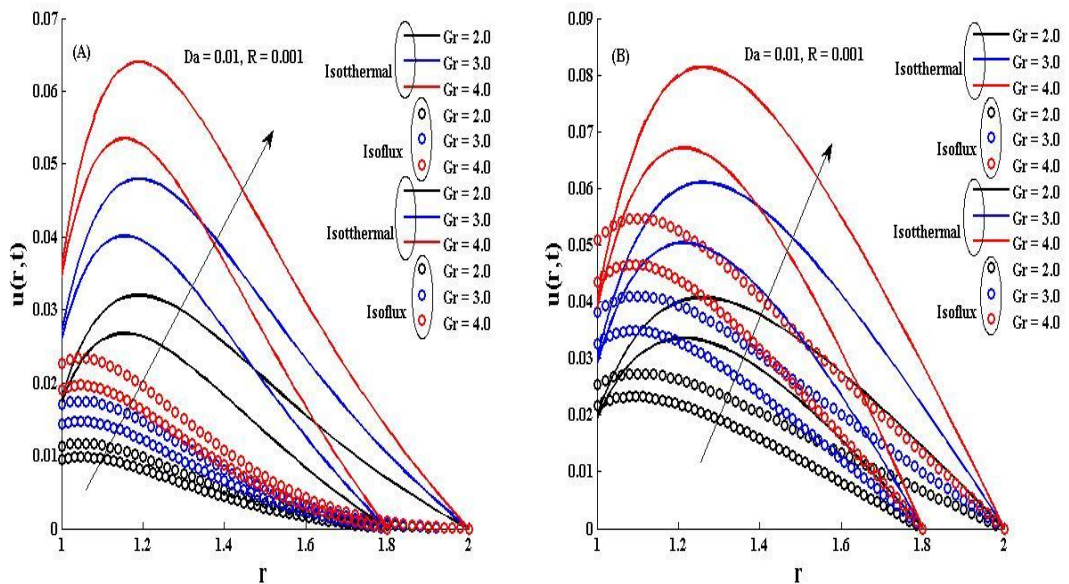


Figure 5 presents a side view of velocity, highlighting the variation in values for Grashof number, suction (A), and injection (B)

Figure 5 illustrates how changes in the Grashof number affect the velocity profile. Figure 5(A) shows the effect under suction conditions, and Figure 5(B) under injection conditions. In Figure 5(A), it was observed that

increasing the Grashof number leads to an increase in the velocity profile under both isothermal and isoflux conditions, with the effect being more pronounced under isothermal conditions. This is attributed to stronger temperature gradients and buoyancy effects, which drive more vigorous convection currents and enhance overall heat transfer efficiency. Similarly, in Figure 5(B), increasing the Grashof number elevates the velocity profile under both isothermal and isoflux conditions. Comparing Figures 5(A) and 5(B), the increase in velocity is more robust in Figure 5(B) due to stronger buoyancy forces influenced by the Grashof number and the injection parameter.

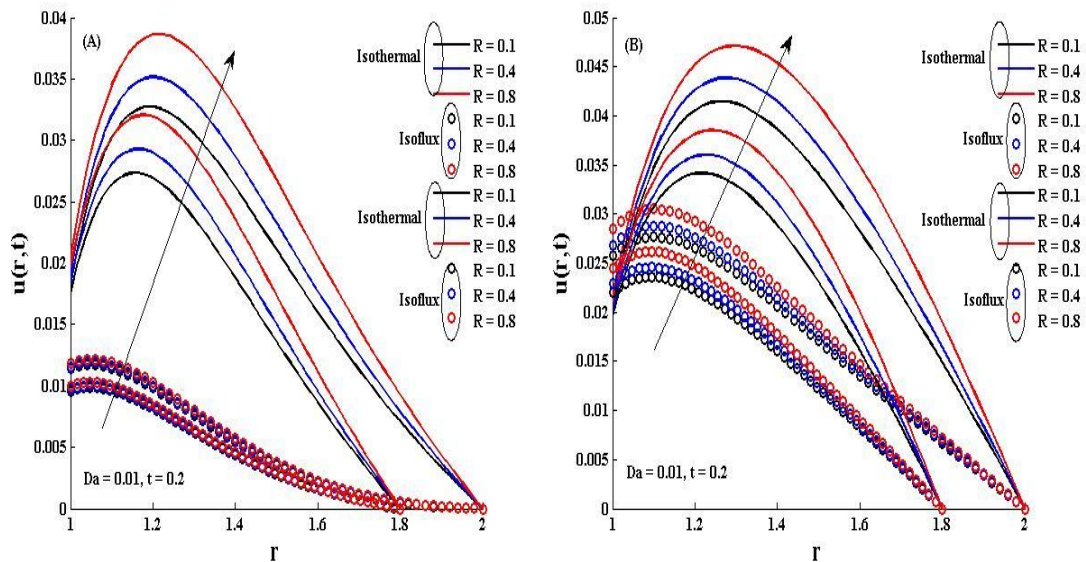


Figure 6 presents a side view of velocity, highlighting the variation in values for Radiation parameter, suction (A), and injection (B)

Figure 6 illustrates the impact of the radiation parameter on the velocity profile, with Figure 6(A) depicting suction and Figure 6(B) showing injection. Notably, increasing the Radiation parameter in Figure 6(A) leads to a rise in the velocity profile under isothermal conditions. However, under isoflux conditions, it results in a stagnating flow with the effect of suction. In Figure 6(B), increasing the radiation parameter causes an upsurge in the velocity profile under both isothermal and isoflux conditions, with the effect being more pronounced under isothermal conditions.

## 7. Temperature profiles

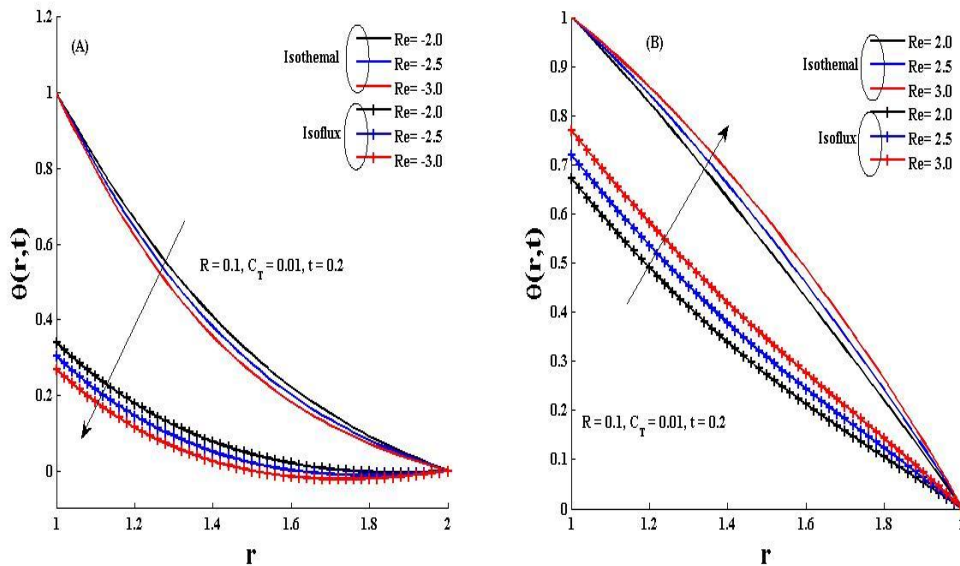


Figure 7 presents a side view of temperature, highlighting the variation in values for suction (A) and injection (B)

Figure 7 illustrates the temperature profiles for variations in the suction parameter 7(A) and the injection parameter 7(B). As shown in Figure 7(A), increasing the suction parameter leads to a decrease in the temperature profile under both isothermal and isoflux conditions. Conversely, Figure 7(B) demonstrates that increasing the injection parameter results in an increase in the temperature profile under both conditions. This is because the injection parameter enhances buoyancy effects, which strengthen convection currents. These enhanced currents improve heat distribution and raise the temperature profile.

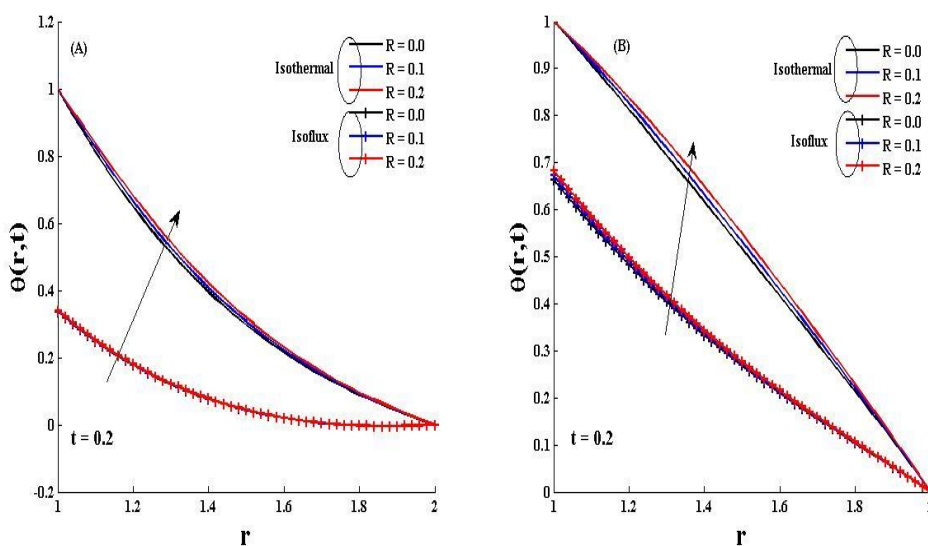


Figure 8 presents a side view of temperature, highlighting the variation in values for Radiation parameter suction (A) and injection (B)

The impact of the radiation parameter  $R$  is depicted in Figures 8(A) suction and 8(B) injection. It is observed that increasing the thermal radiation parameter leads to an increase in the thermal profile in Figures 8(A), under isothermal, however when an electrically conducting fluid is kept under isoflux condition the temperature stagnant at one point. Similarly, in Figures 8(B) the increasing effect was observed under both isothermal and isoflux conditions with more pronounce effect under isothermal condition. This increase is attributed to the rise in the supremacy of conduction of the radiation parameter and the enhancement of the buoyancy effect of the injection parameter to the thermal boundary layer.

**Table 1: Comparison between nonslip Isothermal and Isoflux Conditions of the Unsteady state Skin Friction for the Variation of Suction ( $Re^* < 0$ ) with  $Da = 0.04$**

| $Re^*$ | $\lambda$ | $\tau_1$   |            |         |            | $\tau_\lambda$ |            |         |            |
|--------|-----------|------------|------------|---------|------------|----------------|------------|---------|------------|
|        |           | Isothermal |            | Isoflux |            | Isothermal     |            | Isoflux |            |
|        |           | $R = 0$    | $R = 0.01$ | $R = 0$ | $R = 0.01$ | $R = 0$        | $R = 0.01$ | $R = 0$ | $R = 0.01$ |
| -2.0   | 1.2       | 0.0433     | 0.0434     | 0.0033  | 0.0033     | 0.0915         | 0.0917     | 0.0156  | 0.0156     |
|        | 1.4       | 0.1162     | 0.1164     | 0.0134  | 0.0134     | 0.1004         | 0.1006     | 0.0276  | 0.0276     |
|        | 1.6       | 0.1905     | 0.1909     | 0.0226  | 0.0226     | 0.0983         | 0.0985     | 0.0268  | 0.0268     |
|        | 1.8       | 0.2544     | 0.2548     | 0.0268  | 0.0268     | 0.0895         | 0.0898     | 0.0171  | 0.0172     |
| -2.5   | 1.2       | 0.0426     | 0.0427     | 0.0031  | 0.0031     | 0.0878         | 0.0880     | 0.0143  | 0.0143     |
|        | 1.4       | 0.1140     | 0.1142     | 0.0118  | 0.0118     | 0.0938         | 0.0941     | 0.0233  | 0.0233     |
|        | 1.6       | 0.1866     | 0.1870     | 0.0192  | 0.0193     | 0.0899         | 0.0902     | 0.0209  | 0.0209     |
|        | 1.8       | 0.2493     | 0.2498     | 0.0221  | 0.0221     | 0.0806         | 0.0808     | 0.0112  | 0.0112     |
| -3.0   | 1.2       | 0.0419     | 0.0420     | 0.0028  | 0.0028     | 0.0842         | 0.0844     | 0.0130  | 0.0130     |
|        | 1.4       | 0.1116     | 0.1118     | 0.0103  | 0.0103     | 0.0876         | 0.0878     | 0.0194  | 0.0194     |
|        | 1.6       | 0.1825     | 0.1829     | 0.0160  | 0.0161     | 0.0821         | 0.0823     | 0.0155  | 0.0155     |
|        | 1.8       | 0.2440     | 0.2445     | 0.0176  | 0.0176     | 0.0724         | 0.0726     | 0.0059  | 0.0060     |

**Table 2: Comparison between nonslip Isothermal and Isoflux Conditions of the Unsteady state Skin Friction for the Variation of Injection ( $Re^* > 0$ ) with  $Da = 0.04$**

| $Re^*$ | $\lambda$ | $\tau_1$   |            |         |            | $\tau_\lambda$ |            |         |            |
|--------|-----------|------------|------------|---------|------------|----------------|------------|---------|------------|
|        |           | Isothermal |            | Isoflux |            | Isothermal     |            | Isoflux |            |
|        |           | $R = 0$    | $R = 0.01$ | $R = 0$ | $R = 0.01$ | $R = 0$        | $R = 0.01$ | $R = 0$ | $R = 0.01$ |
| 2.0    | 1.2       | 0.0488     | 0.0489     | 0.0058  | 0.0058     | 0.1251         | 0.1254     | 0.0309  | 0.0309     |
|        | 1.4       | 0.1316     | 0.1318     | 0.0307  | 0.0307     | 0.1630         | 0.1634     | 0.0838  | 0.0838     |
|        | 1.6       | 0.2138     | 0.2141     | 0.0565  | 0.0566     | 0.1831         | 0.1835     | 0.1052  | 0.1054     |
|        | 1.8       | 0.2823     | 0.2827     | 0.0698  | 0.0699     | 0.1862         | 0.1867     | 0.0937  | 0.0938     |
| 2.5    | 1.2       | 0.0495     | 0.0495     | 0.0062  | 0.0062     | 0.1298         | 0.1300     | 0.0335  | 0.0335     |
|        | 1.4       | 0.1331     | 0.1333     | 0.0335  | 0.0335     | 0.1719         | 0.1723     | 0.0943  | 0.0943     |
|        | 1.6       | 0.2155     | 0.2157     | 0.0616  | 0.0616     | 0.1956         | 0.1961     | 0.1198  | 0.1199     |
|        | 1.8       | 0.2839     | 0.2843     | 0.0758  | 0.0758     | 0.2013         | 0.2018     | 0.1077  | 0.1078     |
| 3.0    | 1.2       | 0.0501     | 0.0502     | 0.0066  | 0.0066     | 0.1345         | 0.1348     | 0.0363  | 0.0363     |
|        | 1.4       | 0.1344     | 0.1346     | 0.0364  | 0.0365     | 0.1810         | 0.1815     | 0.1057  | 0.1058     |
|        | 1.6       | 0.2168     | 0.2171     | 0.0668  | 0.0669     | 0.2085         | 0.2090     | 0.1356  | 0.1358     |
|        | 1.8       | 0.2850     | 0.2854     | 0.0818  | 0.0819     | 0.2170         | 0.2176     | 0.1229  | 0.1230     |

Table 1 compares the isothermal and isoflux conditions for the unsteady-state skin friction due to variations in suction ( $Re^* < 0$ ) with  $Da = 0.04$  at the outer surface of the inner cylinder and the inner surface of the outer cylinder, both in the presence and absence of radiation parameters. It is observed that there is decrease in skin friction on all surfaces of the cylinder under both isothermal and isoflux conditions. Under isothermal conditions, skin friction is higher in the presence of the radiation parameter compared to its absence. However, under isoflux conditions, skin friction remains the same regardless of the presence or absence of the radiation parameter.

Table 2 compares the isothermal and isoflux conditions for unsteady-state skin friction due to variations in injection parameter ( $Re^* > 0$ ) with  $Da = 0.04$  at the outer surface of the inner cylinder and the inner surface of the outer cylinder, both in the presence and absence of radiation parameters. An increase in the injection parameter leads to higher skin friction at both surfaces under both isothermal and isoflux conditions. However, under isothermal conditions, skin friction is greater with the presence of the radiation parameter compared to its absence. Under isoflux conditions, skin friction remains mostly unchanged across the space, with only negligible differences at certain points.

## 8. Conclusion

The analysis of the MHD-free convective flow with thermal Rosseland diffusion approximation and porous material in an isothermal and isoflux condition within a coaxial cylinder has been conducted, and the research arrived at the following conclusion:

1. Increase in the Darcy parameter, Radiation parameter, Grashof number results in more rapid increase in velocity near the boundary
2. Increasing the Magnetic and suction parameters results in a reduction in the velocity profile under both isothermal and isoflux conditions.
3. Increasing the suction parameter leads to a decrease in the temperature profile under both isothermal and isoflux conditions. Conversely, increasing the injection parameter results in an increase in the temperature profile
4. Increasing the thermal radiation parameter leads to an increase in the thermal profile in Figures 8(A), under isothermal, however, under isoflux condition the temperature stagnant at one point
5. Under isothermal conditions, increase in suction parameter, leads to a decrease in skin friction on all surfaces of the cylinder with a higher effect in the presence of the radiation parameter compared to its absence. However, under isoflux conditions, skin friction remains the same regardless of the presence or absence of the radiation parameter.
6. An increase in the injection parameter leads to higher skin friction at both surfaces, under isothermal conditions, skin friction is greater with the presence of the radiation parameter compared to its absence. Under isoflux conditions, skin friction remains mostly unchanged across the space, with only negligible differences at certain points.

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| NOMENCLATURE |                                                 | GREEK SYMBOLS |                                      |
|--------------|-------------------------------------------------|---------------|--------------------------------------|
| SYMBOL       | DESCRIPTION                                     | SYMBOL        | DESCRIPTION                          |
| $a$          | Radius of the inner cylinder (m)                | $\alpha$      | Thermal diffusivity                  |
| $b$          | Radius of the outer cylinder (m)                | $\beta$       | Coefficient of thermal expansion     |
| $B_0$        | Constant magnetic heat flux                     | $\eta$        | Slip parameter                       |
| $g$          | Acceleration due to gravity (m/s <sup>2</sup> ) | $\lambda$     | Radii ratio ( $b/a$ )                |
| $Gr$         | Grashof number                                  | $\mu_e$       | Magnetic permeability                |
| $k$          | Thermal conductivity of the fluid (W/mK)        | $\nu$         | Fluid kinematic viscosity            |
| $M$          | Hartmann number                                 | $\theta$      | Dimensionless temperature            |
| $q$          | Heat flux                                       | $\tau$        | Skin friction                        |
| $r'$         | Dimensional radial coordinate                   | $\sigma$      | Stefan-Bolzman constant              |
| $r$          | Dimensionless radial coordinate                 | $\rho$        | Fluid density                        |
| $R$          | Thermal radiation parameter                     | $\sigma_1$    | Electrical conductivity of the fluid |
| $Re$         | Suction/Injection parameter                     |               |                                      |
| $T'$         | Dimensional temperature                         |               |                                      |
| $T_0$        | Initial temperature                             |               |                                      |
| $T_w$        | Temperature of the hot cylinder                 |               |                                      |
| $u'$         | Axial velocity (m/s)                            |               |                                      |
| $U$          | Dimensionless axial velocity                    |               |                                      |
| $z'$         | Vertical coordinate                             |               |                                      |
| $C_T$        | Temperature difference parameter                |               |                                      |
| $K^*$        | The permeability of the porous material         |               |                                      |
| $a'$         | Cross sectional Area                            |               |                                      |
| $L$          | Length of pressure drop                         |               |                                      |
| $K$          | Thermal conductivity                            |               |                                      |

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