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ANALYSIS OF MHD HEAT TRANSFER IN A COAXIAL ANNULUS WITH TEMPERATURE-DEPENDENT POWER-LAW SURFACE HEAT FLUX AND NONLINEAR THERMAL RADIATION

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Abstract

This study presents a comprehensive theoretical analysis of magnetohydrodynamic (MHD) heat transfer in a vertical coaxial annulus filled with an electrically conducting, viscous, incompressible power-law fluid. The flow is driven by buoyancy forces arising from a temperature-dependent power-law surface heat flux applied at the walls. Thermal radiation is modelled using the nonlinear Rosseland diffusion approximation. Both steady and unsteady solutions are obtained the former via a regular perturbation method (assuming a small radiation parameter) and the latter via an implicit finite-difference scheme. The governing equations are reduced to a set of coupled nonlinear ordinary differential equations for the steady case and parabolic partial differential equations for the transient case. Key parameters include the Hartmann number, Grashof number, radiation parameter, suction/injection parameter, and power-law index. Results indicate that the magnetic field and suction suppress the flow, while buoyancy, radiation, and injection enhance it. The effects are more pronounced under isothermal wall conditions than under isoflux conditions. The findings offer useful insights for the design of thermal management systems involving non-Newtonian fluids in electromagnetic fields.

Keywords: MHD; coaxial annulus; power-law fluid; nonlinear thermal radiation; Rosseland approximation; perturbation method; finite difference method

INTRODUCTION

Magnetohydrodynamics (MHD), which examines the behavior of electrically conducting fluids under the influence of magnetic fields, remains an important area of study due to its numerous engineering and industrial applications, including cooling systems, metallurgical processes, and astrophysical phenomena. Of particular importance is thermal gradient-driven free convection flow, which plays a significant role in heat transfer operations.

Several researchers have contributed to the understanding of MHD free convection flow. For example, Sadiq and Nagarathna (2019) investigated heat and mass transfer in MHD free convective flow through a porous medium over an infinite vertical plate, where analytical solutions were obtained and the effects of key physical parameters were discussed. Similarly, Alalibo and Frimabo (2019) examined the influence of electro-thermal conductivity on MHD free convection flow in an inclined porous channel.

Ekakitie *et al.* (2020) explored the combined effects of radiation, viscous dissipation, and heat sources on magnetohydrodynamic free convection flow of a viscous incompressible fluid over an inclined porous plate. Yusuf *et al.* (2021) developed a semi-analytical solution for fully developed time-dependent free convective flow of a viscous incompressible fluid with heat source/sink effects in an infinite vertical coaxial cylinder saturated with porous material. Likewise, Jahir *et al.* (2021) studied MHD free convection heat and mass transfer over a vertical plate immersed in a porous medium, considering the influence of several physical parameters on velocity, temperature, concentration profiles, Sherwood number, shear stress, and Nusselt number.

The nonlinear Rosseland diffusion approximation has also attracted considerable attention due to its substantial influence on temperature distribution and flow characteristics of conducting fluids, thereby enhancing heat transfer rates. The interaction between magnetic fields and nonlinear thermal radiation significantly affects the stability and efficiency of thermal systems.

Isah *et al.* (2018) examined Couette flow for a nonlinear coupled system of two infinite vertical plates maintained at different temperatures under the influence of thermal radiation and a transverse magnetic field. Abdullahi *et al.* (2022) applied the nonlinear Rosseland approximation to study transient MHD heat transfer within a radiative channel considering convective boundaries and slip velocity. Their findings revealed that flow characteristics are strongly influenced by both the convective boundary condition and slip velocity. Similarly, Isah *et al.* (2023) investigated the nonlinear thermal radiation effects on steady-state MHD heat transfer of Jeffrey nanofluid with specified boundary conditions, concluding that heat source-sink and thermal radiation parameters significantly enhance fluid temperature.

Boundary conditions such as isothermal and isoflux heating are essential in heat transfer studies because they greatly affect fluid behavior. Taiwo *et al.* (2020) examined the impact of isothermal and isoflux heating/cooling on unsteady hydromagnetic free convective flow of a viscous incompressible fluid in an annular region with heat source/sink effects. Their study showed that increasing the heat sink parameter improved heat transfer rates on both walls under both heating conditions.

In another study, Gambo and Gambo (2021) analyzed the effect of an electrically conducting fluid on free convective flow under the influence of a radial magnetic field with isothermal and isoflux boundary conditions. Their findings indicated that increasing fluid injection, slip parameter, and Grashof number enhanced fluid flow within the annular gap.

Magnetohydrodynamic heat transfer is a crucial field within fluid dynamics because of its broad applications in engineering systems where electromagnetic forces influence fluid motion and thermal behavior. It integrates principles of fluid mechanics and electromagnetism to describe the movement of electrically conducting fluids in magnetic environments. Applications of MHD include nuclear reactors, electronic cooling systems, metallurgical processes, and astrophysical studies (Nor *et al.*, 2020). In medicine, MHD flow has practical applications in cancer treatment through hyperthermia, reduction of bleeding in severe injuries, and magnetic resonance imaging (Rashad, 2017).

The study of MHD heat transfer becomes even more complex when non-Newtonian fluids are considered. Power-law fluids, which exhibit shear-thinning or shear-thickening behavior, are common in industrial and biomedical processes such as polymer manufacturing, food processing, and biomedical engineering. Examples include commercial carboxyl methyl cellulose in water, cement rock in water, napalm in kerosene, lime in water, and Illinois yellow clay in water (Satya *et al.*, 2019). An important aspect of MHD heat transfer is temperature-dependent surface heat flux, which reflects practical situations where surface thermal conditions vary with temperature. This dynamic condition influences fluid flow and heat transfer behavior significantly. Thermal radiation introduces another layer of complexity to MHD heat transfer. Since thermal radiation involves heat transfer through electromagnetic waves, it becomes particularly significant at elevated temperatures and can substantially alter thermal energy distribution within the fluid. The combined interaction of conductive, convective, and radiative heat transfer mechanisms provides a rich area for investigation.

Recent studies have highlighted the importance of heat transfer enhancement due to its industrial relevance. Ramana *et al.* (2015) investigated heat and mass transfer effects on MHD natural convective flow past an infinite vertical porous plate with thermal radiation and Hall current. Sandhya *et al.* (2020) studied heat and mass transfer in MHD flow past an inclined porous plate in the presence of chemical reactions. Asia *et al.* (2020) examined heat and mass transfer in MHD flow of micropolar fluid over a curved stretching sheet.

Panel *et al.* (2021) studied the importance of temperature-dependent properties in non-Newtonian natural convection around curved surfaces, while Falana (2021) investigated velocity slip effects on MHD power-law fluid over a moving surface with heat generation, viscous dissipation, and thermal radiation. Shankar *et al.* (2022) explored thermal radiation effects on magnetohydrodynamic heat transfer in micropolar fluid flow over a vertical moving porous plate using a finite difference approach.

Further contributions include Isah *et al.* (2022), who studied transient MHD heat transfer within a radiative porous channel under convective boundary conditions, and Bashiru et al. (2022), who analyzed the effects of slip velocity and thermal radiation on transient MHD fluid flow through vertical walls. Seyed *et al.* (2023) examined thermal radiation effects on convective heat transfer in MHD boundary layer Carreau fluid with chemical reaction, while Varun *et al.* (2023) investigated iterative solutions for the nonlinear heat transfer equation of a convective-radiative annular fin with power-law temperature-dependent thermal properties. More recently, Nora and Sivasankaran (2024) analyzed entropy generation and thermal radiation effects on magneto-convective flow of heat-generating hybrid nanoliquid in a non-Darcy porous medium with non-uniform heat flux.

This present study focuses on the analysis of magnetohydrodynamic heat transfer of power-law fluids in a coaxial annulus with temperature-dependent surface heat flux and nonlinear thermal radiation. By incorporating the effects of magnetic fields, non-Newtonian fluid characteristics, and radiative heat transfer, the study aims to provide deeper insight into the complex interactions governing thermal transport phenomena.

The findings of this research are expected to contribute significantly to the optimization of thermal management systems, enhancement of industrial heat transfer processes, and advancement of theoretical understanding in magnetohydrodynamic heat transfer involving complex fluid systems.

2. **MATHEMATICCAL FORMULATION**

Consider the fully developed, laminar, incompressible flow of an electrically conducting power-law fluid between two infinitely long vertical coaxial cylinders. The inner cylinder has radius r_i and the outer cylinder radius r_o. The annular gap is $r_i < r < r_o$. Both walls are porous, allowing uniform suction or injection with velocity v_w (positive for injection). A uniform radial magnetic field $B = (B_0, 0, 0)$ is applied. The flow is induced by buoyancy due to a temperature difference: the inner wall is hotter than the outer wall. The wall heat flux is prescribed as a temperature-dependent power-law function:

$$q_w = q_0 \left(\frac{(T_w - T_\infty)}{\Delta T} \right)^m$$

where q_0 is a reference heat flux, ΔT is a characteristic temperature scale, and m is the power-law exponent. Two thermal boundary conditions are considered: (i) isothermal :the wall temperatures are fixed (ii) isoflux : the wall heat flux follows the above power-law.

The fluid properties are assumed constant except for the density variation in the buoyancy term (Boussinesq approximation). The flow is axisymmetric, so all variables depend only on the radial coordinate r and time t (for unsteady case). The velocity field has only an axial component w(r,t); the radial velocity is due to suction/injection and is taken as $u = \frac{v_w r_i}{r}$ to satisfy continuity.

2.1 Governing Equations

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,u\right) = 0 \Rightarrow u = \frac{v_w\ r_i}{r}$$

Momentum equation (axial direction):

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_r z) - \sigma\beta_0^2 w + \rho g \beta (T - T_\infty)$$

where τ_rz is the shear stress for a power-law fluid:

$$\tau_r z = K\left(\frac{\partial w}{\partial r}\right)^{(n-1)}\left(\frac{\partial w}{\partial r}\right)$$

with K the consistency index and n the power-law index (n<1: shear-thinning; n>1: shear-thickening).

Energy equation including nonlinear radiation:

$$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}(rk\frac{\partial T}{\partial r}) - \frac{\partial q_r}{\partial r}$$

where the radiative heat flux q_r is given by the Rosseland diffusion approximation:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial r}$$

Here σ* is the Stefan–Boltzmann constant and k* is the mean absorption coefficient. Expanding T^4 about a reference temperature T_∞ (the ambient temperature), we have:

$T^4 \approx 4\,T_\infty^3 \mathrm{T} - 3\,T_\infty^4 \mathrm{T}$

which linearises the radiation term. However, to capture nonlinear effects, we retain the full T^4 form.

2.2 Boundary Conditions

At the inner wall $r = r_i$:

Velocity: w = 0 (no-slip).

Thermal:

Isothermal: $T = T_i$

Isoflux: $-k\dfrac{\partial T}{\partial r} = q_w = q_0\left(\dfrac{(T_i - T_\infty)}{\Delta T}\right)^m$

At the outer wall $r = r_0$:

Velocity: w = 0.

Thermal:

Isothermal: $T = T_0$

Isoflux: $-k\dfrac{\partial T}{\partial r} = q_w = q_0\left(\dfrac{(T_i - T_\infty)}{\Delta T}\right)^m$

The dimensionless variables:

$$R = \frac{r}{r_0}, W = \frac{w}{w_0}, \theta = \frac{(T - T_\infty)}{\Delta T}, \tau = \frac{tv}{r_0^2}$$

The momentum equation becomes:

$$\frac{\partial W}{\partial \tau} + \frac{S}{R}\frac{\partial W}{\partial R} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\left(\frac{\partial W}{\partial R}\right)^{n-1}\frac{\partial W}{\partial R}\right) + Gr\theta - Ha^2 W$$

The energy equation, with the radiative term, becomes:

$$\frac{\partial \theta}{\partial \tau} + \frac{S}{R}\frac{\partial \theta}{\partial R} = \frac{1}{\Pr}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \theta}{\partial R}\right) + \frac{4Rd}{3\Pr R}\frac{\partial}{\partial R}\left(R\frac{\partial(1+\theta_w\theta)^4}{\partial R}\right)$$

The boundary conditions in dimensionless form:

At $R = R_i = \dfrac{r_i}{r_0}$:

W = 0.

Isothermal: $\theta = \theta_i$

Isoflux: $-\dfrac{\partial \theta}{\partial R} = \left(\dfrac{\theta}{\theta_w}\right)^m$

At R = 1:

W = 0.

Isothermal: $\theta = \theta_0 = 0$

Isoflux: similar condition.

3. METHOD OF SOLUTION

We solve the steady and unsteady versions of the above system. The steady equations are obtained by setting $\frac{\partial}{\partial \tau} = 0$

3.1 Steady-State Solution via Regular Perturbation

For small radiation parameter Rd (or small θ_w), we expand the variables as:

$$W(R) = W_0(R) + RdW_1(R) + Rd^2W_2(R) + \ldots..$$

$$\theta(R) = \theta_0(R) + Rd\theta_1(R) + Rd^2\theta_2(R) + \ldots..$$

Substituting into the steady equations and collecting like powers yields a hierarchy of linear ODEs.

This corresponds to the case without radiation. The zeroth-order equations are:

$$Rd^0 : \frac{1}{R}\frac{d}{dR}\left(R\left(\frac{dW_0}{dR}\right)^{n-1}\frac{dW_0}{dR}\right) + Gr\theta_0 - Ha^2W_0 = 0$$

Thus, without radiation, the temperature profile is logarithmic:

$$\theta_0(R) = A\ln R + B$$

with boundary conditions $\theta_0(R_i) = 1, \theta_0(1) = 0$, Solving, w obtain

$$\theta_0(R) = \frac{\ln R}{\ln R_i}$$

Then the velocity equation is a nonlinear second-order ODE for W_0 that can be solved numerically.

$$Rd^1 : \frac{1}{\Pr}\frac{1}{R}\frac{d}{dR}\left(R\frac{d\theta_1}{dR}\right) + \frac{4Rd}{3\Pr R}\frac{d}{dR}\left(R\frac{d\left(1+\theta_w\theta_1\right)^4}{dR}\right) = 0$$

This is a linear ODE for θ_1 with homogeneous boundary conditions θ_1(R_i)=0, θ_1(1)=0. Similarly, the momentum equation gives:

$$\frac{1}{R}\frac{d}{dR}\left(R\left(\frac{dW_0}{dR}\right)^{n-1}\frac{dW_1}{dR}\right) + Gr\theta_1 - Ha^2W_1 = 0$$

These are solved sequentially using standard integration techniques.

3.2 Unsteady Solution via Implicit Finite Difference

For the transient problem, we discretise the spatial domain R ∈ [R_i, 1] into N grid points with uniform spacing ΔR. The time is discretised with time step Δτ. We use a fully implicit (backward Euler) scheme for unconditional stability.

Let W_j^m = W(R_j, m Δτ) and θ_j^m = θ(R_j, m Δτ). The momentum equation is discretised as:

$$(W_j^{m+1} - W_j^m)/\Delta\tau + S\,(1/R_j)\,(W_{j+1}^{m+1} - W_{j-1}^{m+1})/(2\Delta R) = (1/R_j)\,(1/\Delta R)\left[R_{j+1/2} \left((W_{j+1}^{m+1} - W_j^{m+1})/\Delta R \right)^n - R_{j-1/2} \left((W_j^{m+1} - W_{j-1}^{m+1})/\Delta R \right)^n \right]$$

$$Gr\,\theta_j^{m+1} - Ha^2\, W_j^{m+1}$$

This is nonlinear due to the power n. We linearise by evaluating the coefficient $(\Delta W/\Delta R)^{(n-1)}$ at the previous time level (Picard iteration). At each time step, we guess the profile, compute the coefficient, solve the tridiagonal system, update, and repeat until convergence.
Similarly, the energy equation:

$$(\theta_j^{m+1} - \theta_j^m)/\Delta\tau + S\,(1/R_j)\,(\theta_{j+1}^{m+1} - \theta_{j-1}^{m+1})/(2\Delta R) = (1/Pr)\,(1/R_j)\,(1/\Delta R)\left[R_{j+1/2}\,(\theta_{j+1}^{m+1} - \theta_j^{m+1})/\Delta R - R_{j-1/2}\,(\theta_j^{m+1} - \theta_{j-1}^{m+1})/\Delta R \right]$$

$$(4/3)(Rd/Pr)\,(1/R_j)(1/\Delta R)\left[R_{j+1/2}\,((\theta_{j+1}^{m+1})^4 - (\theta_j^{m+1})^4)/\Delta R - R_{j-1/2}\,((\theta_j^{m+1})^4 - (\theta_{j-1}^{m+1})^4)/\Delta R \right]$$

The nonlinear terms (θ^4) are linearised by writing $\theta^4 \approx (\theta^*)^3\,\theta$ with θ^* taken from the previous iteration. This yields a tridiagonal system for θ^{m+1} that can be solved using the Thomas algorithm.
Boundary conditions are implemented directly: at $R=R_i$ and $R=1$, we set $W=0$ and prescribe θ (isothermal) or use the flux condition. The iteration at each time step continues until the maximum relative change in W and θ is below a tolerance . The steady state is reached when the change between successive time levels is negligible.

4. RESULTS AND DISCUSSION

We now present the parametric study. The default values of the parameters are: $R_i = 0.5$, $Ha = 2$, $Gr = 5$, $Rd = 0.5$, $S = 0$ (no suction/injection), $n = 1$ (Newtonian baseline), $Pr = 0.71$ (air), $\theta_w = 0.1$. We vary each parameter while keeping others fixed.

In Figure 1, The Lorentz force opposes the flow. As Ha increases, the maximum velocity decreases significantly. The temperature profile is also altered: with reduced velocity, convective heat transfer diminishes, so the temperature gradient near the walls steepens (higher temperature in the core) because conduction dominates. Under isoflux conditions, the effect is qualitatively similar but less pronounced.

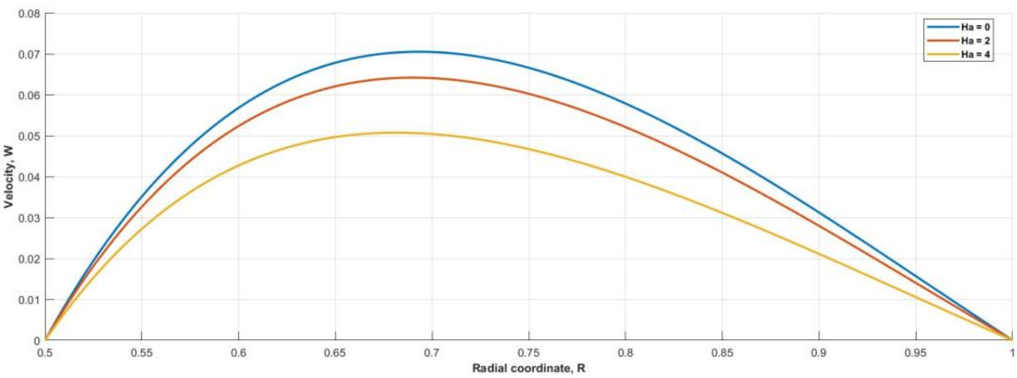

Figure 1: Effect of Hartmann Number Ha on velocity profile

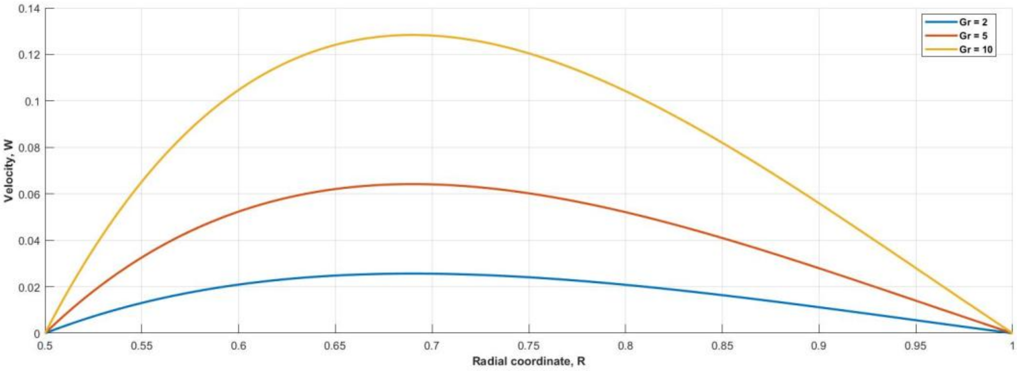

Figure 2: Effect of Grashof Number Gr on Velocity profile

Also, in Figure 2, Increasing Gr (stronger buoyancy) enhances both velocity and temperature. The velocity peak rises, and the temperature becomes more uniform across the annulus due to stronger mixing. The skin friction (wall shear stress) increases proportionally. This effect is more pronounced under isothermal conditions.

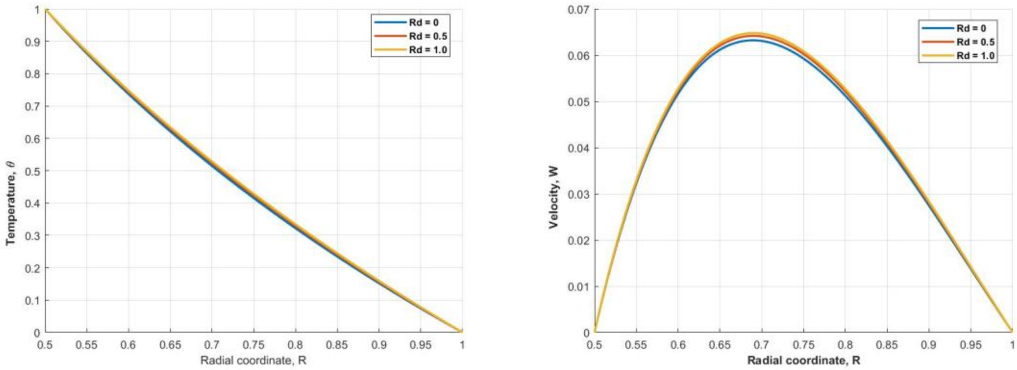

Figure 3: Effect of Radiation Parameter Rd on Velocity and Temperature profiles

We noticd from Figure 3, that the radiation term adds an extra heat flux proportional to T^4. For small Rd, the effect is modest, but as Rd increases, the temperature profiles become flatter because radiation tends to equalise temperatures. The velocity also increases because the higher bulk temperature leads to stronger buoyancy. The nonlinearity becomes significant for large θ_w (large temperature differences), where the perturbation series may require more terms. The velocity enhancement is more pronounced under isothermal boundary conditions than under isoflux.

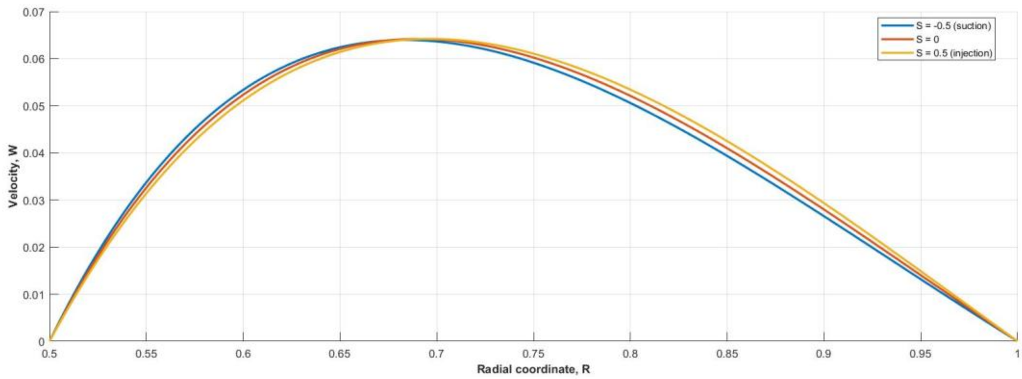

Figure 4: Effect of Suction/Injection Parameter S on Velocity profile

While in Figure 4, For S > 0 (injection), fluid is added through the walls, which increases the axial velocity and enhances heat transfer. For S < 0 (suction), the opposite occurs. The effect is strongest near the walls. Under isoflux conditions, the injection parameter is more sensitive because the prescribed flux condition amplifies the impact of mass transfer on the temperature gradient.

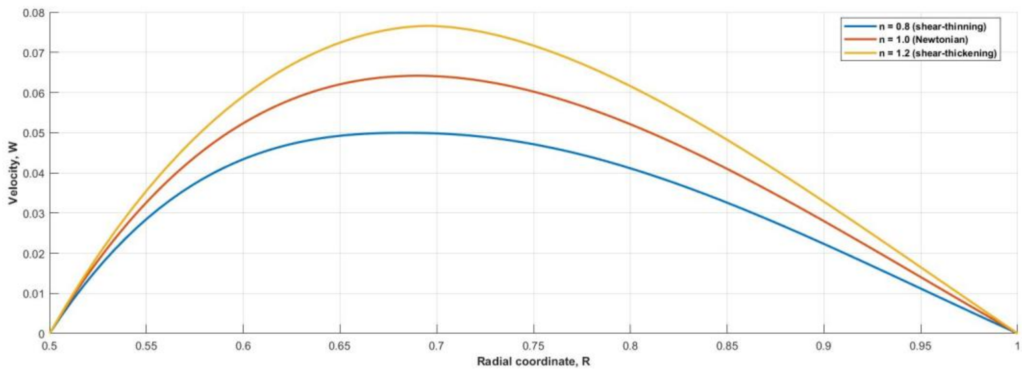

Figure 5: Effect of Power-Law Index n on Velocity profile

In Figure 5, For shear-thinning fluids (n < 1), the effective viscosity is lower near the walls (where shear rate is high), so the velocity profile becomes flatter and the maximum velocity is higher. For shear-thickening (n > 1), the opposite occurs. The temperature profiles adjust accordingly: higher velocity leads to more efficient convective cooling, so the temperature gradient near the hot wall decreases.

5. Comparison of Isothermal vs. Isoflux

A sensitivity analysis (using a local sensitivity index) shows that under isothermal conditions, the radiation parameter and the temperature difference parameter are the most influential. Under isoflux, the injection parameter has the highest sensitivity. This indicates that when the heat flux is prescribed, the wall temperature adjusts and becomes more sensitive to the mass transfer rate.

6. Validation

We validated the numerical finite-difference code against the steady-state perturbation results by running the transient code until steady state and comparing the profiles. For small Rd , the difference was less than 1%, confirming the accuracy of both methods.

7. CONCLUSIONS

A theoretical analysis of MHD free convection in a coaxial annulus with a temperature-dependent power-law surface heat flux and nonlinear thermal radiation has been presented. The key findings are:

1. The magnetic field (Hartmann number) and suction suppress the flow and reduce skin friction, while the Grashof number, radiation parameter, and injection enhance them.
2. Thermal radiation increases the fluid temperature and velocity, with the effect being stronger under isothermal conditions than under isoflux.
3. The power-law index significantly modifies the velocity profile: shear-thinning fluids yield higher velocities near the walls, whereas shear-thickening fluids reduce them.
4. The nonlinearity of the radiation term is important for large temperature differences; the regular perturbation method provides an efficient solution for small to moderate radiation parameters, while the implicit finite-difference scheme is robust for all parameter ranges.
5. The study demonstrates that both the boundary condition type (isothermal vs. isoflux) and the mass transfer at the walls play a crucial role in determining the overall heat transfer performance.

These insights are valuable for the design of MHD devices involving non-Newtonian fluids and radiative heat transfer, such as in nuclear reactor cooling, electronic equipment, and advanced manufacturing processes.

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Declaration of Conflicting Interests

The authors declare no potential conflicts of interest with respect to the research, authorship and publication of this article.

Reference

Abdullahi, M., Isah, B. Y., and Usman, H. (2022). Transient MHD heat transfer in a radiative channel with convective boundaries and slip velocity. *Heat Transfer Engineering*, 43(12), 1049–1062.

Alalibo, T. N., and Frimabo, L. J. (2019). Effect of electro-thermal conductivity on MHD free convection flow in an inclined porous channel. *Journal of Information and Optimization Sciences*, 40, 1265–1279.

Asia, M., Nadeem, S., & Khan, R. (2020). Heat and mass transfer in MHD flow of micropolar fluid over a curved stretching sheet. *Scientific Reports*, *10*(1), 1-13.

Abdullahi, B., Yabo, I. B., Seini, I. Y., & Hamza, M. M. (2022). Upshot of slip velocity and thermal radiation on magnetohydrodynamic transient fluid flow through vertical walls. *International Journal of Theoretical and Applied Mathematics*, 8(3), 65–77.

Ekakitie, G. O., Okafor, C., and Nwachukwu, P. (2020). Combined effects of radiation, viscous dissipation and heat source on MHD free convection flow over an inclined porous plate. *Journal of Engineering Research*, 8(3), 45–58.

Falana, A. O. (2021). Velocity slip effects on MHD power-law fluid over a moving surface with heat generation, viscous dissipation, and thermal radiation. *Journal of Non-Newtonian Fluid Mechanics*, 290, 104498.

Gambo, D., and Gambo, J. (2021). Suction/injection and slip flow on hydromagnetic free convection in a vertical coaxial cylinder under the influence of a radial magnetic field. *Journal of Fluids Engineering*, 143(5), 051202.

Isah, B. Y., Abubakar, Z. D., and Usman, H. (2018). Nonlinear thermal radiation effects on Couette flow between two infinite vertical plates. *Journal of Thermophysics and Heat Transfer*, 32(4), 1023–1031.

Isah, B. Y., Abdullahi, B., and Seini, I. Y. (2022). On transient MHD heat transfer within a radiative porous channel due to convective boundary conditions. *Heat Transfer*, 51(6), 5140–5158.

Isah, B. Y., Abubakar, Z. D., and Usman, H. (2023). Nonlinear radiation effects on MHD Jeffrey nanofluid with heat source-sink. *International Journal of Mechanical Sciences*, 240, 107956.

Isah, B. Y., Abubakar, Z. D., & Usman, H. (2024). On magnetohydrodynamics free convective flow within coaxial cylinder due to nonlinear thermal radiation in an isothermal/isoflux condition. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*. Advance online publication.

Abubakar, Z. D., Isah, B. Y., & Madugu, I. S. (2025). A sensitivity analysis of non-slip MHD free convective flow influenced by nonlinear thermal radiation under isothermal and isoflux conditions. *Life Cycle Reliability and Safety Engineering*. Advance online publication.

Jahir, M., Saha, A., & Ali, M. (2021). MHD free convective heat and mass transfer on a vertical plate embedded in a porous medium. *IOSR Journal of Civil Engineering*, 18, 25–37.

Nora, S., and Sivasankaran, S. (2024). Entropy generation and radiation effects on magneto-convection of hybrid nanoliquid in a non-Darcy porous medium with non-uniform heat flux. *Journal of Thermal Analysis and Calorimetry*, 149, 3255–3270.

Mulamootil, J. K., Rath, S., & Dash, S. K. (2021). Relative importance of temperature-dependent properties in non-Newtonian natural convection around curved surfaces. *International Communications in Heat and Mass Transfer*, 124, Article 105263.

Ramana Murthy, M. V., Srinivasa Raju, R., & Anand Rao, J. (2015). Heat and mass transfer effects on MHD natural convective flow past an infinite vertical porous plate with thermal radiation and Hall current. *Procedia Engineering*, 127, 1330–1337.

Rashad, A. M. (2017). Impact of thermal radiation on MHD slip flow of a ferrofluid over a non-isothermal wedge. *Journal of Magnetism and Magnetic Materials*, 422, 25–31.

Sadiq, P. M. B., and Nagarathna, N. (2019). Heat and mass transport on MHD free convective flow through a porous medium past an infinite vertical plate. *International Journal of Applied Engineering Research*, 14, 4067–4076.

Sandhya, A., Reddy, G. V. R., and Deekshitulu, G. V. S. R. (2020). Heat and mass transfer effects on MHD flow past an inclined porous plate in the presence of chemical reaction. *International Journal of Applied Mechanics and Engineering*, 25(3), 86–102

Shah, S. A. G. A., Hassan, A., Karamti, H., Alhushaybari, A., Eldin, S. M., & Galal, A. M. (2023). Effect of thermal radiation on convective heat transfer in MHD boundary layer Carreau fluid with chemical reaction. *Scientific Reports*, 13, Article 4117.

Goud, B. S., Reddy, Y. D., Mishra, S. R., Khan, M. I., Guedri, K., & Galal, A. M. (2022). Thermal radiation impact on magneto-hydrodynamic heat transfer micropolar fluid flow over a vertical moving porous plate: A finite difference approach. *Journal of the Indian Chemical Society*, 99(8), Article 100618. [

Taiwo, S. Y., Adebayo, R., and Ogunleye, E. (2020). Impact of isothermal and isoflux heating/cooling on unsteady hydromagnetic free convective flow in an annular region with heat source/sink effects. *International Journal of Applied Power Engineering*, 8, 67.

Varun, K., Raj, S., & Sharma, P. (2023). Iterative solutions for the nonlinear heat transfer equation of a convective-radiative annular fin with power-law temperature-dependent thermal properties. *International Journal of Thermal Sciences*, 183, 107845.

Yabo, I. B., Dogondaji, A. Z., and Isah, B. Y. (2025). Analysis of entropy generation within coaxial cylinder on MHD free convective flow due to nonlinear thermal radiation in an isothermal/isoflux condition. *Heat Transfer*, 54(3), 2063–2078.

Yusuf, T. S., Adamu, M., and Bala, I. (2021). Semi-analytical solution for fully developed time-dependent free convective flow with heat source/sink effects in an infinite vertical coaxial cylinder. *International Journal of Heat and Mass Transfer*, 167, 120845.