International Journal of
Science, Architecture, Technology and Environment

# Mathematical Modelling and Numerical Simulation of Time-Dependent Flow of a Viscous Reactive Fluid and their Application

Zayyanu M.*, Dogondaji Z. A., Zayyanu S. Y., Saudat S.

*Department of Mathematics. Sokoto state university, Sokoto, Nigeria*

*Corresponding author*

*DOI: https://doi.org/10.63680/ijsate0726069.73*

## Abstract

This study investigates an unsteady state Magnetohydrodynamic (MHD) free convection flow with the effects of Hartmann number and nonlinear heat generation. The study focuses on the coupled behavior of velocity and temperature fields in the presence of a transverse magnetic field. The governing partial differential equations for momentum and energy are formulated in dimensionless form and solved numerically using a finite difference method. The influence of key physical parameters such as the Hartmann Number (), Prandtl Number (), heat generation parameter (), activation energy parameter (), and thermal diffusion parameter () on velocity and temperature distributions is analyzed. The results shows that the magnetic field significantly suppresses fluid motion, while nonlinear heat generation enhances the temperature field. The study is relevant to engineering and industrial applications involving electrically conducting fluids, such as MHD generators, cooling of nuclear reactors, and metallurgical processes.

***Keywords:*** Magnetohydrodynamics (MHD), Unsteady Free Convection, Hartmann Number, Nonlinear Heat Generation, Finite Difference Method, Heat Transfer, Prandtl Number, Electrically Conducting Fluid.

## 1. Introduction

Fluid flow and heat transfer phenomena play a vital role in many engineering and industrial applications, including power generation, chemical processing, petroleum engineering, and environmental systems. The transport of heat and momentum within fluids has been extensively studied due to its importance in improving system efficiency and thermal performance (Galal *et al.,* 2024). Over the years, advanced mathematical and numerical models have been developed to analyze the complex interactions between velocity, temperature, and other physical quantities in fluid systems (Nabwey *et al.,*2023). One important area of study is the behavior of electrically conducting fluids in the presence of a magnetic field, known as magnetohydrodynamics (MHD). This field has gained significant attention due to its wide range of applications, including electromagnetic pumps, cooling of nuclear reactors, and metallurgical processes (Rashad *et al.,* 2024). When a magnetic field is applied to a conducting fluid, it produces a resistive Lorentz force that affects fluid motion and can be used to control flow behavior in engineering systems (Zaman *et al.*, 2023). In addition to magnetic effects, heat transfer in fluids occurs through conduction, convection, and thermal radiation. Thermal radiation becomes particularly significant in high-temperature environments such as gas turbines and combustion systems, where it contributes substantially to energy transport (Radhikaa and Reddy, 2024). To simplify radiative heat transfer modeling, the Rosseland approximation is commonly used to incorporate radiation effects into the governing energy equations (Zaman *et al.,* 2023).

Another important factor influencing heat transfer is internal heat generation or absorption due to chemical reactions. Many industrial processes involve exothermic or endothermic reactions, and their rates are often temperature-dependent, following Arrhenius-type expressions. This introduces strong nonlinearity into the governing equations and increases the complexity of the analysis (Jeelani et al., 2024). Such effects are particularly important in reactive flows and energy systems (Rasheed *et al.,* 2021). Furthermore, buoyancy effects caused by temperature differences within the fluid play a crucial role in natural convection flows. Variations in temperature lead to density differences, which induce fluid motion under gravitational forces, thereby coupling thermal and flow fields (Ali *et al.,* 2025).

To better analyze these complex interactions, it is common to transform the governing equations into dimensionless form. This approach introduces key dimensionless parameters that characterize the system and help identify the relative importance of physical effects such as viscous forces, magnetic influence, thermal radiation, and chemical reactions (Zaman *et al.,* 2023).

The present study focuses on the combined effects of magnetic field, thermal radiation, and temperature-dependent chemical reaction on fluid flow and heat transfer. These combined effects lead to highly nonlinear behavior, which requires the use of analytical and numerical techniques for proper investigation (Ali *et al.,* 2025; Jeelani *et al.,* 2024).

Understanding such coupled systems is essential for optimizing engineering processes. For example, magnetic fields can be used to control fluid motion, while proper heat transfer management enhances thermal system efficiency. Additionally, incorporating chemical reactions is important for accurately predicting temperature distribution in reactive systems (Galal *et al.,* 2024).

Therefore, this study contributes to the growing body of knowledge by examining the combined influence of magnetic field, radiation, and nonlinear chemical reactions on fluid flow and heat transfer, which has important applications in science and engineering.

Fluid flow and heat transfer phenomena play a vital role in many engineering and industrial applications such as power generation, petroleum engineering, and thermal systems. These processes are governed by the interaction of momentum and energy transport within fluids, which are often modeled using coupled nonlinear partial differential equations (Mahmud *et al.,* 2023).

In the present study, the flow is influenced by a magnetic field, characterized by the Hartmann number (Ha). The application of a magnetic field to an electrically conducting fluid generates a resistive Lorentz force, which opposes fluid motion and reduces velocity. This magnetic damping effect is widely used to control flow behavior in engineering systems (Zahid *et al*., 2023; Abbas *et al.*, 2022).

Heat transfer in the system is governed by thermal diffusion and is influenced by the Prandtl number (Pr), which represents the ratio of momentum diffusivity to thermal diffusivity. Fluids with higher Prandtl numbers exhibit thinner thermal boundary layers, significantly affecting temperature distribution (Zaman *et al.,* 2023). Thermal radiation is another important mechanism incorporated in the model. In high-temperature environments, radiative heat transfer becomes significant and is commonly modeled using the Rosseland diffusion approximation, which simplifies the radiative heat flux term in the energy equation. This approximation is widely used in MHD heat transfer studies (Nityanand *et al.,* 2024; Amer *et al.,* 2024).

The effect of chemical reaction is introduced through the parameter lambda (λ, reaction rate parameter), which represents internal heat generation or absorption within the fluid. In many industrial processes, chemical reactions follow Arrhenius-type behavior, where the reaction rate depends exponentially on temperature. This introduces strong nonlinearity into the governing equations (Jeelani *et al.,* 2024; Abbas *et al.,* 2022).

Additionally, the parameter epsilon (ε, activation energy parameter) further controls the temperature dependence of the reaction term. The exponential form of the reaction term, significantly increases the complexity of the energy equation and affects temperature profiles within the fluid (Zaman *et al.,* 2023).

Buoyancy effects, arising due to temperature differences, also contribute to fluid motion through natural convection. These effects are important in many MHD flow problems where both thermal and momentum fields are strongly coupled (Algehyne *et al.,* 2023). To simplify the analysis, the governing equations are transformed into dimensionless form, introducing key parameters such as Ha, Pr, λ, and ε, which describe the relative importance of magnetic force, thermal diffusion, and chemical reaction. This approach reduces the complexity of the problem and allows for a clearer understanding of the physical behavior of the system (Amer *et al.,* 2024).

The combined effects of magnetic field, thermal radiation, and nonlinear chemical reaction lead to highly nonlinear coupled equations. Analytical and numerical methods are therefore required to investigate the influence of these parameters on velocity and temperature distributions (Magdy *et al.,* 2025). Understanding the influence of these parameters is essential for optimizing engineering processes. For example, increasing the Hartmann number (Ha) can suppress fluid motion, while variations in λ and ε significantly affect heat generation and temperature distribution. Similarly, radiation enhances energy transport in high-temperature systems.

Therefore, this study provides a detailed analysis of the combined effects of Prandtl number (Pr), Hartmann number (Ha), and radiation(R) parameters, contributing to the advancement of research in MHD fluid flow and heat transfer.

## 1. Mathematical Formulation

Consider the unsteady, laminar, one-dimensional flow of an incompressible, electrically conducting Newtonian fluid along a vertical channel. The flow configuration consists of two infinite vertical parallel plates separated by a distance $h$, with the $x$-axis oriented vertically upward along the channel and the $y$-axis measured normal to the plates. A uniform transverse magnetic field of strength $B_0$ is applied perpendicular to the plates (i.e., along the $y$-direction). The magnetic Reynolds number is assumed to be very small, so that the induced magnetic field is negligible compared to the applied magnetic field.

The fluid motion is driven by buoyancy forces arising from temperature and concentration differences, with the plates maintained at constant temperature $T_w$ and constant species concentration $C_w$, which are higher than the corresponding ambient values $T_0$ and $C_0$ far from the plates. Internal heat generation occurs within the fluid due to a chemical reaction following Arrhenius kinetics, and thermal radiation effects are incorporated using the Rosseland approximation. Viscous dissipation is assumed negligible, and all fluid properties are considered constant except for density variations in the buoyancy terms, which are handled via the Boussinesq approximation.

## 2. Mathematical Analysis

Under the above assumptions, the governing equations for momentum and energy are expressed in dimensional form as:

$$\frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T(T^* - T_0) - \frac{\sigma B_0^2}{\rho} u^* \qquad 1$$

$$\frac{\partial T^*}{\partial t^*} = \frac{\alpha}{\rho Cp} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q_r}{\partial y^*} + \frac{QC_0A}{\rho Cp} e^{\frac{-E}{RT^*}} \qquad 2$$

Subject to initial and boundary conditions

$$\left.\begin{matrix} u^* = 0, & T^* = T_1 \\ u^* = 0, & T^* = T_2 \end{matrix}\right\} \qquad 3$$

Using the Rosseland approximation, the radiative heat flux is given as:

$$q_r = \frac{16\sigma T_\infty^3}{3kk^*} \qquad 4$$

Substituting this into the energy equation simplifies the radiative heat flux term into a conduction-like form.

To reduce the governing equations 1 – 2 into dimensionless form, the following nondimensional variables are introduced:

$$y^* = Hy, \; t^* = \frac{H^2}{\nu} t, \; u^* = \frac{g\beta H^2 RT_0^2 u}{\nu E}, \; \frac{QRT_0^2}{E} = T^* - T_0 \qquad 5$$

Substituting these variables into the governing equations and simplifying yields the dimensionless equations.

The initial and boundary conditions in dimensionless form is:

$$\begin{cases} u = 0, \theta = 1, C = 1 & \text{at } y = 0, \\ u = 0, \theta = 0, C = 0 & \text{at } y = 1. \end{cases} \qquad 6$$

After the introduction of the dimensionless variables and parameters, the governing equations 1 – 2 becomes:

Momentum Equation

$$\frac{\partial u}{\partial t} = \frac{d^2 u}{dy^2} + \theta - H_a^2 u \qquad 7$$

Energy Equation

$$\Pr \frac{\partial \theta}{\partial t} = \frac{Pr}{a_1} \frac{\partial^2 \theta}{\partial y^2} + \lambda e^{\frac{\theta}{1+\varepsilon\theta}} \qquad 8$$

## 3. Numerical solution (Finite Difference Method FDM)

Equation 7 – 8 is discretized numerically using an implicit finite difference method over a uniform grid in space and time to obtain:

Momentum Equation

$$-r_1 U_{i-1}^{j+1} + (1+2r_1)U_i^{j+1} - r_1 U_{i+1}^{j+1} = (1-\Delta t H_a^2)U_i^j + \Delta t \theta_i^j \quad 9$$

Energy Equation

$$-r_2 \theta_{i-1}^{j+1} + (1+2r_2)\theta_i^{j+1} - r_2 \theta_{i+1}^{j+1} = \theta_i^j + \frac{\Delta t \lambda}{Pr} e^{\frac{\theta_i^j}{1+\varepsilon\theta_i^j}} \quad 10$$

Where: $a_1 = \frac{3Pr}{3+4RPr}$; $r_1 = \frac{\Delta t}{(\Delta y)^2}$; $r_2 = \frac{\Delta t}{a_1(\Delta y)^2}$

### 4.1 Skin Friction, and Nusselt Number Evaluations

The skin friction, and Nusselt number are important physical quantities that describe the rates of momentum, and heat transfer at the boundaries. These quantities depend on the gradients of velocity, and temperature at the wall, respectively.

At the walls (y = 0 and 1), a second-order accurate forward difference scheme is employed.

At the lower wall (y = 0)

$$SK_0 = \frac{-3\text{vold}(1) + 4vold(2) - vold(3)}{2\Delta y} \quad 11$$

$$NU_0 = \frac{-3told(1) + 4told(2) - told(3)}{2\Delta y} \quad 12$$

At the upper wall (y = 1)

$$SK_1 = \frac{3vold(end) - 4vold(m) + vold(m-1)}{2\Delta y} \quad 13$$

$$NU_1 = \frac{3told(end) - 4\text{told}(\text{m}) + told(m-1)}{2\Delta y} \quad 14$$

## 4. Analytical solution (Perturbation Technique)

Due to the nonlinear nature of the governing equations, exact analytical solutions are difficult to obtain. Therefore, an approximate analytical method based on perturbation technique is employed.

The analysis is carried out under steady-state conditions; hence, all time-dependent terms are neglected. i.e

$$\frac{\partial u}{\partial t} = 0, \frac{\partial \theta}{\partial t} = 0 \quad 15$$

Applying equation (3.15), equation (3.7) – (3.8) becomes:

Momentum Equation

$$\frac{d^2u}{dy^2} + \theta - H_a^2 u = 0 \quad 16$$

Energy Equation

$$\frac{1}{a_1}\frac{\partial^2\theta}{\partial y^2}+\frac{\lambda}{Pr}e^{\frac{\theta}{1+\varepsilon\theta}} = 0 \qquad 17$$

In this study, lambda ($\lambda$) is considered as the perturbation parameter. A general solution is assumed:

$$\left.\begin{aligned} u(y) &= u_0(y)+\lambda u_1(y) \\ \theta(y) &= \theta_0(y)+\lambda\theta_1(y) \end{aligned}\right\} \qquad 18$$

Applying appropriate boundary conditions and substituting equation 18 into equation 16 – 17 respectively to obtain the following:

$$u_0(y) = A_1e^{Hay} + A_2e^{-Hay} + A_3 + A_4y \qquad 19$$

$$u_1(y) = B_1e^{Hay} + B_2e^{-Hay} + B_3y^5 + B_4y^4 + B_5y^3 + B_6y^2 + B_7y + B_8 \qquad 20$$

$$\theta_0(y) = k_1\gamma + k_2 \qquad 21$$

$$\theta_1(y) = -\frac{1}{2}k_3y^2 - \frac{1}{6}k_4y^3 - \frac{1}{12}k_5y^4 - \frac{1}{20}k_6y^5 + k_7y + k_8 \qquad 22$$

Where, $k_1$- $k_8$, $A_1$- $A_2$, $B_1$- $B_6$, and S are all constants introduced to simplify the solving of the equations, $A_3$, $A_4$ $B_7$, and $B_8$ are integrating constants.

## 5. RESULTS AND DISCUSSION

### Introduction

This section presents and discusses the numerical results obtained from the finite difference solution of the governing dimensionless equations for unsteady magnetohydrodynamic (MHD) free convection flow with nonlinear heat generation. The effects of the governing physical parameters, including the Hartmann number (Ha), Prandtl number (Pr), heat generation parameter (λ), activation energy parameter (ε), and thermal diffusion parameter, on the velocity and temperature distributions are examined. The graphical results are interpreted to explain the influence of each parameter on the fluid flow and heat transfer characteristics. The observed trends are also discussed in relation to the underlying physical mechanisms and their engineering significance.

Fig. 1: An increase in the Hartmann number (Ha) results in a reduction in the velocity profile. This occurs due to the influence of the applied magnetic field, which induces a Lorentz force that acts opposite to the direction of fluid motion, thereby inhibiting the flow. Consequently, higher values of Ha enhance the resistive effect and lead to a decrease in fluid velocity.

Fig. 2: An increase in the Prandtl number (Pr) leads to a decline in the temperature profile. This is attributed to the reduction in thermal diffusivity at higher Pr values, which limits heat diffusion within the fluid. Hence, the thermal boundary layer becomes thinner, resulting in a lower temperature distribution.

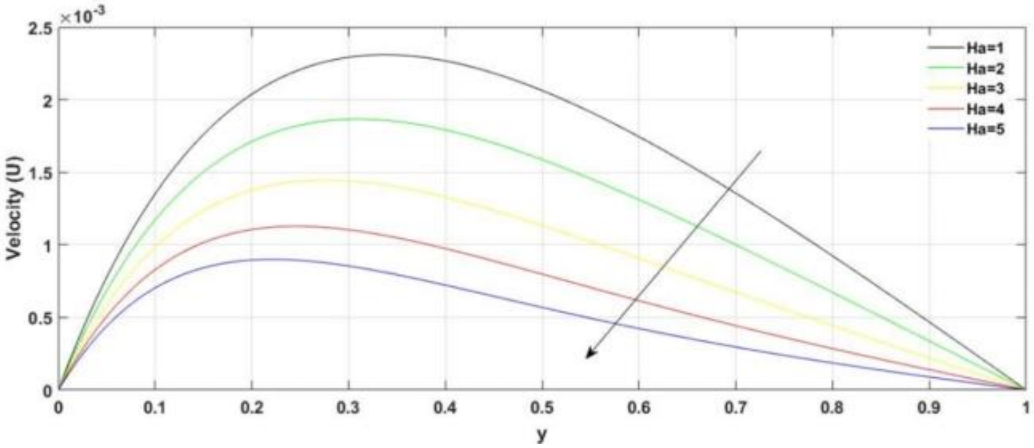


Fig. 1: Effect of Hartmann number (Ha) on velocity profile

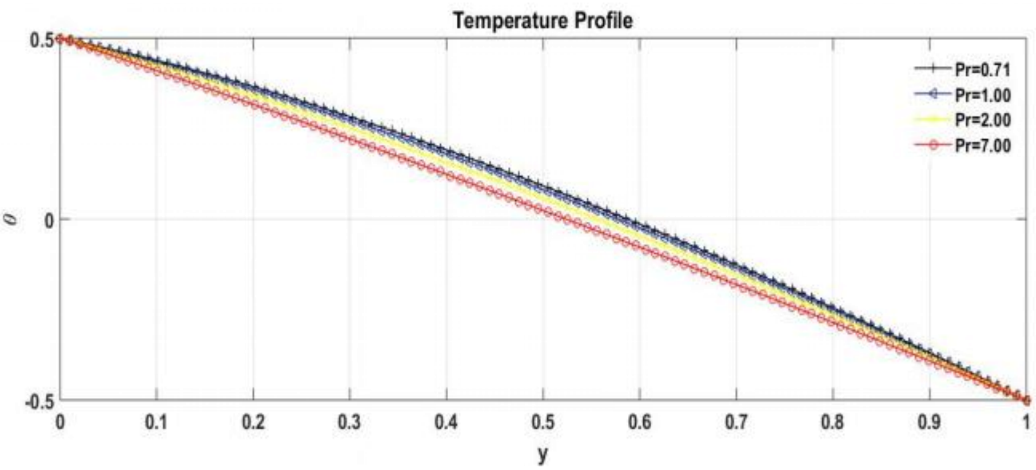


Fig. 2: Effect of Prandtl number (Pr) on temperature profile

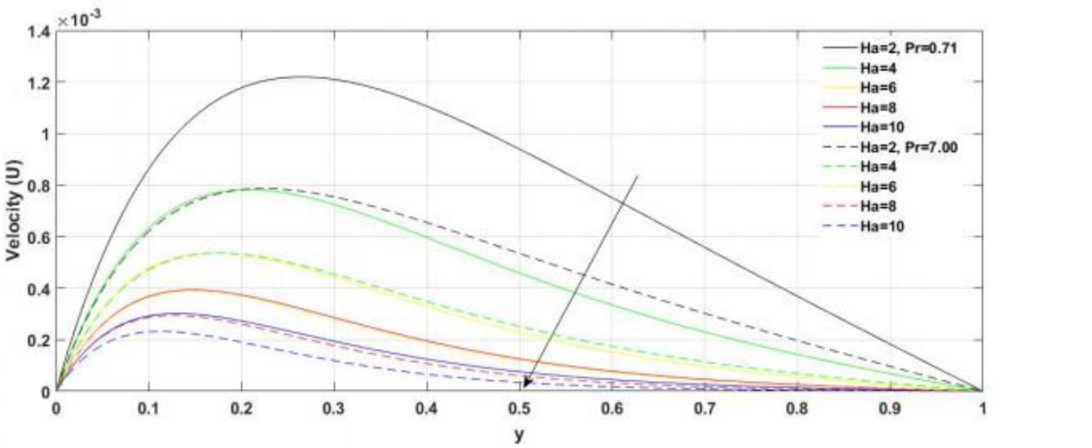


Fig. 3: Effect of Hartmann number (Ha) on velocity at Pr=0.71 and Pr=7.00

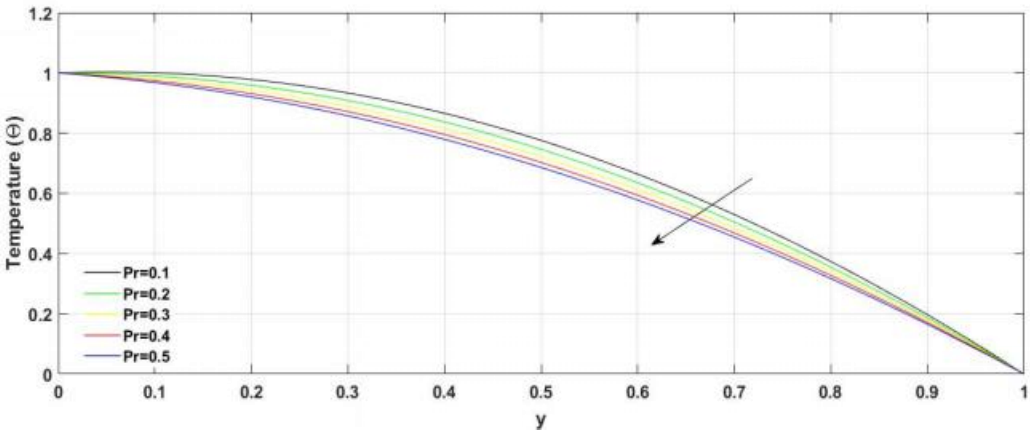


Fig. 4: Effect of Prandtl number (Pr) on temperature profile

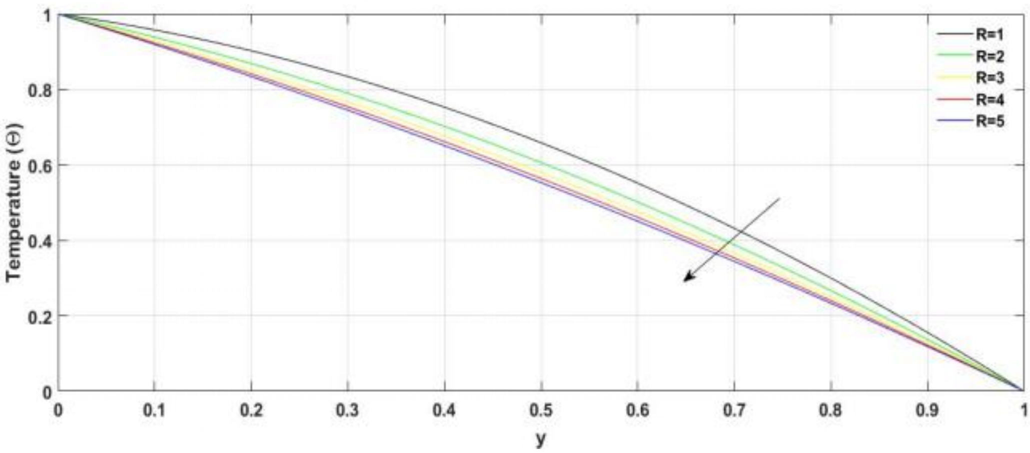


Fig. 5: Effect of Radiation (R) on temperature profile

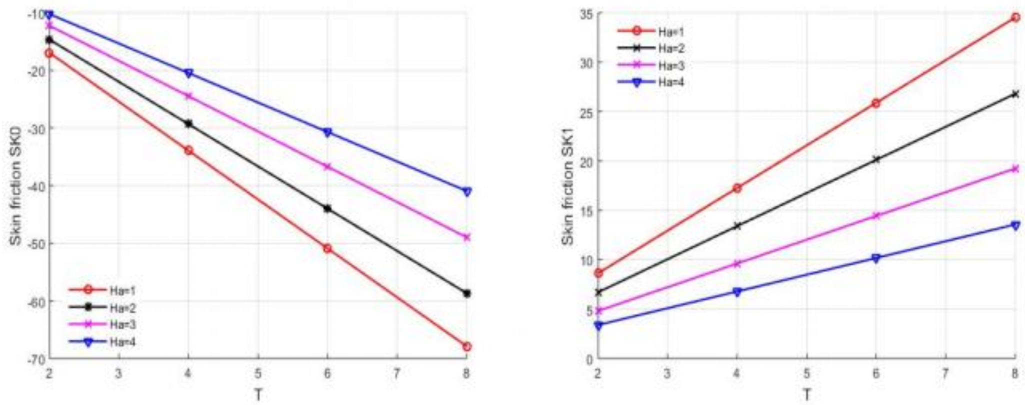


Fig. 6: Variation of skin friction coefficient with Hartmann number (Ha)

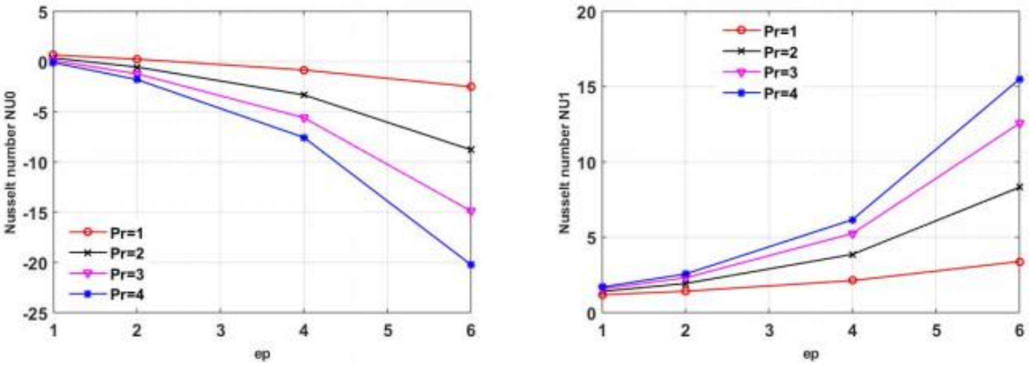


Fig. 7: Variation of Nusselt number coefficient with Prandtl number (Pr)

It is observed from Figs. 3 and 5 that an increase in the Hartmann number (Ha) leads to a reduction in the velocity profile. This is due to the Lorentz force generated by the applied magnetic field, which opposes fluid motion and suppresses velocity. In contrast, Figs. 4 and 6 show that increasing temperature (T) enhances the velocity profile, indicating that higher temperature promotes fluid motion and reduces resistance. The combined effects in Figs. 5 and 6 further show that higher Prandtl number (Pr = 7) results in lower velocity compared to Pr = 0.71, confirming that larger Pr values suppress fluid motion. However, increasing temperature enhances velocity in both air (Pr = 0.71) and water (Pr = 7), though the effect is more pronounced in air.

It is observed from Fig. 7 that increasing the Prandtl number (Pr) reduces the temperature profile due to lower thermal diffusivity, which weakens heat diffusion and reduces the thermal boundary layer thickness. From Fig. 8, increasing the radiation parameter (R) reduces the temperature distribution, indicating enhanced radiative heat loss from the fluid. Fig. 9 shows that increasing λ enhances the temperature profile, suggesting stronger heat retention and thicker thermal boundary layers. Fig. 10 reveals that increasing the heat generation parameter (ε) intensifies thermal gradients within the fluid. The combined effects in Figs. 11 and 12 show that both radiation (R) and heat generation (ε) significantly influence the temperature field under different Hartmann numbers (Ha = 1 and Ha = 10). In both cases, higher Ha leads to smoother and more gradual temperature variations, indicating the influence of the magnetic field on thermal transport.

Fig. 13 shows that increasing the Hartmann number (Ha) reduces the magnitude of skin friction, indicating that the magnetic field suppresses wall shear stress through the Lorentz damping effect.

Fig. 14 shows that the Nusselt number increases with increasing Prandtl number (Pr), indicating enhanced heat transfer at the boundary due to stronger thermal diffusion effects.

## Conclusion

This study examined the unsteady magnetohydrodynamic (MHD) free convection flow with nonlinear heat generation in the presence of a transverse magnetic field. The governing equations were solved numerically using the finite difference method to investigate how different physical parameters influence the flow and heat transfer characteristics.

The results showed that the Hartmann number has a significant damping effect on the fluid velocity because the applied magnetic field opposes the motion of the electrically conducting fluid. On the other hand, increasing the heat generation parameter raises the fluid temperature by supplying additional thermal energy to the system. The Prandtl number, activation energy parameter, and thermal diffusion parameter were also found to influence the temperature distribution, demonstrating their important roles in the heat transfer process.

Overall, the findings of this study improve the understanding of MHD free convection flows with nonlinear heat generation and provide useful information for the design and optimization of engineering systems involving electrically conducting fluids, such as cooling systems, metallurgical processes, and MHD power generation.

## Declaration of Conflicting Interests

The authors declare no potential conflicts of interest with respect to the research, authorship and publication of this article.

## Funding

The author received no financial support for the research, authorship and publication of this article.

## References

Abbas, Z., Sheikh, M. and Iqbal, Z., (2022). Heat transfer in MHD flow with chemical reaction. International Communications in Heat and Mass Transfer, 135, p.106085.

Algehyne (2023). These effects are important in MHD flow problems where both thermal and momentum field are strongly coupled, Heat and mass transfer on MHD nanofluid flow with chemical reaction. American Journal of Nano Research and Applications, 12(2), pp.45–58.

Ali, A. R., Siddig, N. H., Irshad, S., Abu-Nab, A. K., and Majeed, A. H. (2025). Magnetized double-flow with thermal radiation effects. Journal of Nonlinear Mathematical Physics, 32, 37.

Ali, A.R., Siddig, N.H. and Irshad, S., (2025). Magnetized double-diffusive flow with radiation. Journal of Nonlinear Mathematical Physics, 32, p.37.

Amer, (2024) the complenty of problems and allowed for clear understanding of the physical behavior of the system. Processes, 12(6), 1136.

Jeelani, M., Abbas, A. and Alqahtani, N.A., (2024). Chemically reacting MHD hybrid nanofluid with radiation. Processes, 12(6), p.1196.

Galal, A. M., Alharbi, F. M., Arshad, M., Alam, M. M., and Abdeljawad, T. (2024). Numerical investigation of heat and mass transfer in three-dimensional MHD nanoliquid flow. Scientific Reports, 14, 1207.

Jeelani (2024), Abbas (2022). Arrhenius types behavior, where the reaction rate depends exponentially on temperature. Processes, 12(6), 1166.

Jeelani, M. B., Abbas, A., & Alqahtani, N. A. (2024). Thermal transportation in heat-generating and chemically reacting MHD hybrid nanofluid flow. Processes, 12(6), 1196.

Magdy (2025). The influence of these parameters on velocity and temperature distribution.

Mahmud (2023), the interaction of momentum energy transfer withing fluid which are often modeled. 5th ed. Springer.

Nabwey, H. A., Rashad, A. M., Khan, W. A., El-Kabeir, S. M. M., & AbdElnaem, S. (2023). Heat transfer in MHD flow of ternary hybrid nanofluid. Frontiers in Physics, 11, 1212715.

Nityanand (2024), Amer (2024). The approximation is widly used in MHD heat transfer studies. 5th ed. Springer.

Radhikaa, M. and Reddy, Y.D., 2024. Thermal radiation effects on hybrid nanofluid MHD flow. Radiation Effects and Defects in Solids, 179(11–12), pp.1656–1682.

Radhikaa, M., & Reddy, Y. D. (2024). Thermal radiation and heat source effects on MHD nanofluid flow. Radiation Effects and Defects in Solids.

Rashad, A. M., Kolsi, L., Mansour, M. A., Salah, T., Mir, A., & Alshammari, B. M. (2024). Effect of thermal radiation on unsteady MHD nanofluid flow. Frontiers in Chemistry, 12, 1441077.

Rashad, A.M., Kolsi, L. and Mansour, M.A., 2024. Thermal radiation effects in MHD nanofluid flow. Frontiers in Chemistry, 12, p.1441077.

Rasheed, H. U., Islam, S., Helmi, M. M., Alsallami, S. A. M., & Khan, Z. (2021). Internal heating and chemical reaction effects on MHD nanofluid flow. Crystals, 11(12), 1523.

Zahid (2023), and Abbas (2022), effect of widely used to control flow behavior in engineering system. Arrhenius activation energy effects in heat transfer. Journal of Thermal Science and Engineering Applications, 15(2), p.021015.

Zaman (2023), fluid with higher prandial number exhibit thinner thermal boundary layers significally effect temperature distributions. . SAGE Open Engineering.

Zaman, S. U., Aslam, M. N., & Hussain, A. (2023). Chemically reactive MHD fluid flow with thermophoresis and Brownian motion. SAGE Open Engineering.