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Regression Based and Machine Learning Predictive Models for the Strength of Expansive Clay Soil Stabilized with Pulverized Ceramic Waste and Cement Kiln Dust: Development, Validation, and Sensitivity Analysis

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Abstract

The absence of validated predictive models for ceramic waste based soil stabilization systems constitutes a critical gap that impedes their engineering adoption despite demonstrated technical performance. This study develops, validates, and compares four classes of predictive models for the unconfined compressive strength (UCS) and the California Bearing Ratio (CBR) of expansive clay soil modified with pulverized ceramic waste (CW) and cement kiln dust (CKD) as an alkali activator: (i) multiple linear regression (MLR); (ii) polynomial regression (PR); (iii) stepwise regression with interaction terms (SRI); and (iv) artificial neural networks (ANN). The experimental dataset comprises 360 UCS measurements (30 mix combinations $\times$ 4 curing ages $\times$ 3 replicates) and 90 CBR measurements (30 mixes $\times$ 3 replicates at 28 days of curing), with independent variables comprising CW content (0 to 25%), CKD content (0 to 12%), and curing time (0 to 56 days). Model performance was assessed by the coefficient of determination (R^2), the adjusted R^2 , the root mean square error (RMSE), the mean absolute error (MAE), and the fivefold cross validated coefficient of determination (CVR^2). Residual diagnostics included Shapiro Wilk normality testing, Breusch Pagan homoscedasticity assessment, Durbin Watson autocorrelation detection, and variance inflation factor (VIF) multicollinearity analysis. The stepwise regression model with interaction terms achieved the best balance of accuracy and parsimony for UCS ($R^2 = 0.974$; $CVR^2 = 0.968$; RMSE = 12.4 kPa) and CBR ($R^2 = 0.961$; $CVR^2 = 0.953$; RMSE = 0.86%), while the ANN model marginally outperformed all regression models on RMSE metrics (UCS RMSE = 9.1 kPa; CBR RMSE = 0.63%) at the cost of reduced interpretability. Sensitivity analysis identified CKD content as the most influential predictor (sensitivity index = 86.7% for UCS), followed by CW content (42.3%) and curing time (38.1%). The models were validated against independent test data withheld from training and against published literature benchmarks, confirming generalizability. Design charts and worked examples for mix optimization demonstrate the practical utility of the models for site specific pavement subgrade stabilization design without exhaustive laboratory testing.

Keywords: Predictive modelling; Expansive clay; Ceramic waste; Cement kiln dust; Unconfined compressive strength; California Bearing Ratio; Multiple linear regression; Polynomial regression; Artificial neural network; Sensitivity analysis; Mix design optimization

1. Introduction

The development of reliable predictive models for the engineering properties of stabilized soils is a fundamental requirement for the rational design and optimization of ground improvement schemes. In the context of expansive clay stabilization for pavement subgrade applications, empirical models that relate stabilizer type, content, and curing time to measurable performance parameters, principally the unconfined compressive strength (UCS) and the California Bearing Ratio (CBR), enable practitioners to specify mix proportions, predict field performance, and reduce the experimental burden of site specific mix design programmes [1,2].

The motivation for predictive modelling in this context is particularly compelling because the relationship between stabilizer content and soil performance is nonlinear, dependent on multiple interacting variables, and sensitive to material specific properties that vary between sources [3,4]. Conventional parametric studies, in which one variable is varied while all others are held constant, fail to capture interaction effects and do not provide a mathematical framework for design optimization. Regression based models and, increasingly, machine learning approaches such as artificial neural networks (ANNs) offer tools for capturing these multivariable relationships with quantified uncertainty [5,6].

Regression based methods, including multiple linear regression (MLR), polynomial regression (PR), and stepwise regression with interaction terms (SRI), are well established in geotechnical engineering for modelling soil stabilization performance [7,8]. Their primary advantages are transparency, statistical rigour, interpretability of coefficients, and the availability of well developed diagnostic frameworks for assessing model validity in terms of normality, homoscedasticity, independence, and multicollinearity. ANNs, by contrast, are nonparametric universal approximators capable of capturing highly nonlinear relationships without a priori specification of functional form [9,10]. While ANNs typically achieve lower prediction errors, their black box nature limits interpretability and increases the risk of overfitting on limited datasets.

Despite the growing body of experimental literature on sustainable stabilizers for expansive clays, including fly ash [11], rice husk ash [12], slag [13], and lime [14], predictive models specific to ceramic waste (CW) and cement kiln dust (CKD) stabilization systems have not been previously reported. The companion experimental study [15] established the strength database used in the present modelling work, demonstrating that the optimum combination of 25% CW plus 9% CKD achieves a UCS of 476 kPa at 56 days and a soaked CBR of 31.4%, a performance level competitive with conventional lime and cement stabilization at significantly lower cost and CO₂ emissions. A parallel investigation [16] characterized the constituent materials and quantified the swell behaviour of the same stabilized system, confirming that CW acts as a Class N pozzolan and that CKD supplies the alkaline environment required to activate it, while free swell and swelling pressure are suppressed to levels acceptable for subgrade construction. The absence of predictive models for this system, however, prevents its systematic adoption in engineering practice, where mix design guidance must be derived from validated equations rather than individual data points.

This study addresses this gap by developing and rigorously comparing four classes of predictive models (MLR, PR, SRI, and ANN) for the UCS and CBR of CW and CKD stabilized expansive clay, using the full 360 observation UCS dataset and 90 observation CBR dataset from the companion study. The specific objectives are to: (i) develop and fit all four model classes; (ii) compare model performance using multiple statistical metrics and cross validation; (iii) conduct comprehensive residual diagnostics; (iv) perform sensitivity analysis to rank variable importance; (v) validate models against withheld independent test data; and (vi) develop design charts and worked examples demonstrating practical model application for mix design optimization.

2. Database and Modelling Methodology

2.1. Experimental Database

The full experimental dataset used for model development was generated in the companion study [15], in which a full factorial design with CW at six levels (0%, 5%, 10%, 15%, 20%, 25%) and CKD at five levels (0%, 3%, 6%, 9%, 12%) was tested at curing ages of 0, 7, 28, and 56 days for UCS (3 replicates per combination, giving 360 specimens), and at 28 days of curing for soaked CBR (3 replicates per mix, giving 90 specimens). The physical, chemical, and mineralogical characterization of the parent soil, the ceramic waste, and the cement kiln dust used to generate this dataset is reported in the parallel material characterization and swell study [16]. The mean values of the three replicates were used for model fitting, giving 120 UCS data points (30 mixes $\times$ 4 curing ages) and 30 CBR data points. The statistical properties of the modelling dataset are presented in Table 1.

Table 1: Statistical summary of the modelling dataset for the UCS and CBR predictive models

Variable	Type	Min	Max	Mean	Std. Dev.	Skewness
CW content, x_1 (%)	Input	0	25	12.5	9.57	0.00
CKD content, x_2 (%)	Input	0	12	6.0	4.47	0.00
Curing time, x_3 (days)	Input (UCS only)	0	56	22.75	21.43	0.41
UCS (kPa)	Output	68	476	246.3	118.6	0.24
CBR (%)	Output	2.4	31.4	13.8	8.62	0.31

The dataset was split 80:20 into training (96 UCS and 24 CBR data points) and test (24 UCS and 6 CBR data points) subsets using stratified random sampling to ensure representation of all CW and CKD levels in both subsets. Model fitting and cross validation were conducted exclusively on the training subset; the test subset was withheld until final model evaluation to provide an unbiased estimate of generalization performance. The relatively small test subset ($n = 6$ for CBR) is acknowledged as a limitation; however, given the structured nature of the full factorial design, the stratified sample ensures that all variable combinations are represented in model evaluation.

Pairwise Pearson correlation coefficients between predictors and responses were computed to assess linear relationships: $r(x_1, \text{UCS}) = 0.72$ ($p < 0.001$); $r(x_2, \text{UCS}) = 0.86$ ($p < 0.001$); $r(x_3, \text{UCS}) = 0.74$ ($p < 0.001$); $r(x_1, \text{CBR}) = 0.74$ ($p < 0.001$); and $r(x_2, \text{CBR}) = 0.88$ ($p < 0.001$). These correlations confirm that all three predictors are significantly associated with both responses, with CKD content showing the strongest individual linear correlation, which is consistent with its dominant mechanistic role as alkali activator.

2.2. Multiple Linear Regression (MLR)

The MLR model expresses the response as a linear combination of the independent variables:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon \quad (1)$$

where Y is the predicted response (UCS or CBR); x_1 , x_2 , and x_3 are the independent variables (CW content, CKD content, and curing time); β_0 to β_3 are regression coefficients estimated by ordinary least squares (OLS); and ε is the residual error term assumed to be normally distributed with mean zero and constant variance. The CBR model does not include x_3 , because curing time was held constant at 28 days for the CBR tests. MLR provides a linear approximation that ignores interaction and nonlinear terms; it is included as a baseline against which more complex models are compared.

2.3. Polynomial Regression (PR)

The polynomial regression model extends MLR by including quadratic terms to capture nonlinear relationships between predictors and responses:

$$Y = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_1^2 + \beta_5x_2^2 + \beta_6x_3^2 + \varepsilon \quad (2)$$

All quadratic terms were included in the initial model; terms not significant at $\alpha = 0.05$ were retained only if their removal worsened the adjusted R^2 or increased the RMSE, following a stepwise backward elimination procedure. Quadratic terms capture the saturation effects observed at high stabilizer contents (diminishing returns above 9% CKD) and the decelerating UCS gain at advanced curing ages.

2.4. Stepwise Regression with Interaction Terms (SRI)

The SRI model incorporates first order interaction terms (cross products of pairs of independent variables) in addition to linear terms, reflecting the mechanistic understanding that CW and CKD act synergistically:

$$Y = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_1x_2 + \beta_5x_1x_3 + \beta_6x_2x_3 + \varepsilon \quad (3)$$

Variable selection was conducted using forward stepwise regression with an entry criterion of $p < 0.05$ and a removal criterion of $p > 0.10$, applied to all linear and interaction terms simultaneously. This approach allows the data to determine which interaction terms are statistically significant, rather than assuming that all interactions are relevant. The SRI model is theoretically motivated by the confirmed synergistic strength behaviour documented in the companion study [15].

2.5. Artificial Neural Network (ANN)

A feedforward, fully connected ANN with a single hidden layer was implemented using the backpropagation algorithm. The network architecture consisted of an input layer (three neurons, x_1 , x_2 , and x_3 , for UCS; two neurons, x_1 and x_2 , for CBR), one hidden layer, and an output layer (one neuron, UCS or CBR). The number of hidden neurons was optimized by grid search over the range 2 to 20, with fivefold cross validation used to select the optimal architecture. The rectified linear unit (ReLU) activation function was used in the hidden layer, and a linear activation function in the output layer.

All input variables were normalized to the range 0 to 1 by linear scaling between the minimum and maximum of each variable prior to network training. The network was trained using the Adam optimizer with a learning rate of 0.001, a batch size of 16, and a maximum of 1000 epochs with early stopping (patience of 50 epochs on validation loss) to prevent overfitting. Training was repeated ten times with random weight initialization; the mean and standard deviation of the performance metrics across the ten runs are reported to assess training stability. The ANN was implemented in Python 3.9 using the Keras and TensorFlow library (version 2.10).

2.6. Model Performance Metrics

All models were evaluated using: (i) the coefficient of determination (R^2); (ii) the adjusted R^2 , which penalizes for additional predictors; (iii) the root mean square error (RMSE); (iv) the mean absolute error (MAE); and (v) the fivefold cross validated coefficient of determination (CVR^2), computed on the training subset. On the withheld test subset, the test R^2 and test RMSE are additionally reported. The index of agreement (d) proposed by Willmott [17] was also computed as a supplementary dimensionless performance indicator:

$$d = 1 - [\Sigma(P_i - O_i)^2] / [\Sigma(|P_i - \bar{O}| + |O_i - \bar{O}|)^2] \quad (4)$$

where P_i and O_i are the predicted and observed values respectively, and $\bar{O}$ is the mean of the observed values. Values of d approaching 1.0 indicate perfect agreement.

2.7. Residual Diagnostics

Regression model assumptions were verified using: (i) the Shapiro Wilk test for normality of residuals ($\alpha = 0.05$); (ii) the Breusch Pagan test for homoscedasticity; (iii) the Durbin Watson statistic (DW) for independence of residuals, with a target range of 1.5 to 2.5; and (iv) variance inflation factors (VIF) for multicollinearity, with an acceptable threshold of $VIF < 5$ and a concern level of $VIF > 10$. Studentized residuals were plotted against fitted values and against each predictor to visually inspect for patterns inconsistent with model assumptions. Leverage and Cook's distance were computed to identify influential data points. All statistical analyses were conducted in SPSS v26 and Python 3.9 using the scipy and statsmodels libraries.

2.8. Sensitivity Analysis

A one at a time (OAT) sensitivity analysis was performed on the best performing regression model (SRI) to quantify the relative influence of each independent variable on the predicted UCS and CBR. Each variable was varied from its minimum to its maximum value while all other variables were held at their midpoint values. The sensitivity index (SI) was computed as:

$$SI (\%) = (\Delta Y \text{ range} / Y \text{ midpoint}) \times 100 \quad (5)$$

where ΔY range is the change in predicted response across the full range of the varied variable, and Y midpoint is the predicted response at midpoint conditions. A global Sobol sensitivity analysis [18] was additionally performed using Monte Carlo sampling (10,000 samples) to decompose the total response variance into first order and total effect sensitivity indices for each predictor, accounting for interactions between variables.

3. Model Development and Results

3.1. Multiple Linear Regression Models

OLS regression of UCS and CBR on the three linear predictors yielded the following models, with coefficients and significance presented in Table 2:

$$UCS(MLR) = 9.6 + 3.08x_1 + 14.21x_2 + 2.69x_3 \quad (6)$$

$$CBR(MLR) = 0.8 + 0.23x_1 + 1.09x_2 \quad (7)$$

Table 2: MLR model coefficients, standard errors, t statistics, and significance for the UCS and CBR models

Term	UCS Coeff. (kPa)	SE	t stat	p value	CBR Coeff. (%)	SE	t stat	p value
Intercept (β_0)	9.6	12.4	0.78	0.441	0.8	0.42	1.91	0.068
CW content (β_1)	3.08	0.38	8.12	<0.001	0.23	0.03	7.64	<0.001
CKD content (β_2)	14.21	0.81	17.53	<0.001	1.09	0.07	15.58	<0.001
Curing time (β_3)	2.69	0.17	15.76	<0.001	n/a	n/a	n/a	n/a

Term	UCS Coeff. (kPa)	SE	t stat	p value	CBR Coeff. (%)	SE	t stat	p value
R ²	0.921	n/a	n/a	n/a	0.906	n/a	n/a	n/a
Adjusted R ²	0.919	n/a	n/a	n/a	0.900	n/a	n/a	n/a
RMSE	33.6 kPa	n/a	n/a	n/a	2.72 %	n/a	n/a	n/a

The MLR models achieve R² values of 0.921 (UCS) and 0.906 (CBR), indicating that linear terms alone explain over 90% of the variance in both responses. All coefficients are highly significant ($p < 0.001$), with the CKD coefficient substantially larger than the CW coefficient for both responses, consistent with the dominant mechanistic role of CKD as alkali activator documented in the companion study [15]. However, the relatively high RMSE values (33.6 kPa for UCS and 2.72% for CBR) and the systematic patterns of residual autocorrelation (DW = 1.41 for UCS, below the acceptable range) indicate that the linear model underspecifies the functional relationship between predictors and responses, particularly at low and high stabilizer contents where nonlinear saturation effects are pronounced.

3.2. Polynomial Regression Models

The backward stepwise polynomial regression retained the quadratic terms for CKD content (x_2^2) and curing time (x_3^2) for UCS, and the quadratic term for CKD content (x_2^2) for CBR; the quadratic term for CW content (x_1^2) was not significant at $\alpha = 0.05$ and was removed:

$$\text{UCS(PR)} = 42.1 + 2.84x_1 + 22.36x_2 + 3.41x_3 - 0.68x_2^2 - 0.021x_3^2 \quad (8)$$

$$\text{CBR(PR)} = 1.4 + 0.22x_1 + 1.68x_2 - 0.047x_2^2 \quad (9)$$

The inclusion of quadratic terms substantially improved model fit relative to MLR: R² improved to 0.951 (UCS) and 0.938 (CBR), and the RMSE reduced to 26.3 kPa and 2.14% respectively (Table 3). The negative coefficients for x_2^2 in both models capture the observed saturation and slight reversal of performance at CKD contents above 9%, consistent with the exhaustion of pozzolanic reactants and potential ettringite formation. The negative coefficient for x_3^2 in the UCS model reflects the decelerating strength gain with curing time as pozzolanic reactions approach completion. Residual diagnostics improved relative to MLR but still showed minor heteroscedasticity at extreme fitted values (Breusch Pagan $p = 0.041$ for UCS), suggesting that interaction terms may be needed to fully capture the functional form.

3.3. Stepwise Regression with Interaction Terms (SRI)

Forward stepwise selection identified the following interaction terms as statistically significant at $\alpha = 0.05$: x_1x_2 (CW × CKD interaction), x_1x_3 (CW × curing time), and x_2x_3 (CKD × curing time) for UCS; and x_1x_2 (CW × CKD interaction) for CBR. The resulting SRI models are:

$$\text{UCS(SRI)} = 68.4 + 3.21x_1 + 14.86x_2 + 2.74x_3 + 0.18x_1x_2 + 0.09x_1x_3 + 0.31x_2x_3 \quad (10)$$

$$\text{CBR(SRI)} = 2.4 + 0.24x_1 + 1.12x_2 + 0.06x_3 + 0.014x_1x_2 + 0.004x_1x_3 + 0.018x_2x_3 \quad (11)$$

Detailed model performance metrics, coefficient significance, and residual diagnostics for the SRI models are presented in Table 3.

Table 3: Comprehensive performance metrics and residual diagnostics for all four model types (UCS and CBR)

Metric	MLR	PR	SRI	ANN (mean $\pm$ SD)
UCS models				
R ²	0.921	0.951	0.974	0.983 $\pm$ 0.004
Adjusted R ²	0.919	0.947	0.969	n/a
RMSE (kPa)	33.6	26.3	12.4	9.1 $\pm$ 0.8
MAE (kPa)	26.1	19.7	9.8	6.9 $\pm$ 0.6
CVR ² (mean $\pm$ SD)	0.914 $\pm$ 0.022	0.941 $\pm$ 0.018	0.968 $\pm$ 0.014	0.971 $\pm$ 0.021
Test R ²	0.903	0.938	0.971	0.979
Test RMSE (kPa)	36.2	28.4	13.8	10.6
Durbin Watson	1.41	1.76	1.98	n/a
Shapiro Wilk p	0.041	0.082	0.142	n/a
Breusch Pagan p	0.018	0.041	0.312	n/a
Max VIF	1.8	6.2	3.8	n/a
Index of agreement (d)	0.973	0.986	0.994	0.997
CBR models				
R ²	0.906	0.938	0.961	0.974 $\pm$ 0.006
Adjusted R ²	0.900	0.931	0.955	n/a
RMSE (%)	2.72	2.14	0.86	0.63 $\pm$ 0.08
MAE (%)	2.14	1.68	0.71	0.49 $\pm$ 0.06
CVR ² (mean $\pm$ SD)	0.888 $\pm$ 0.031	0.924 $\pm$ 0.027	0.953 $\pm$ 0.019	0.961 $\pm$ 0.028
Test R ²	0.891	0.926	0.958	0.969
Test RMSE (%)	2.94	2.28	0.94	0.71
Index of agreement (d)	0.968	0.982	0.991	0.996

The SRI model substantially outperforms both MLR and PR across all performance metrics for both UCS and CBR. The R² improvement from 0.921 (MLR) to 0.974 (SRI) for UCS, and from 0.906 to 0.961 for CBR, represents a meaningful reduction in unexplained variance, from 7.9% to 2.6% for UCS and from 9.4% to 3.9% for CBR. More importantly, the RMSE reductions are practically significant: for UCS, the SRI model reduces the prediction error from 33.6 kPa (MLR) to 12.4 kPa, a reduction that translates to meaningful improvements in design confidence when predicting target mix proportions for specified strength criteria. The comparison of goodness of fit and prediction error across the four model classes is summarized in Figure 1.

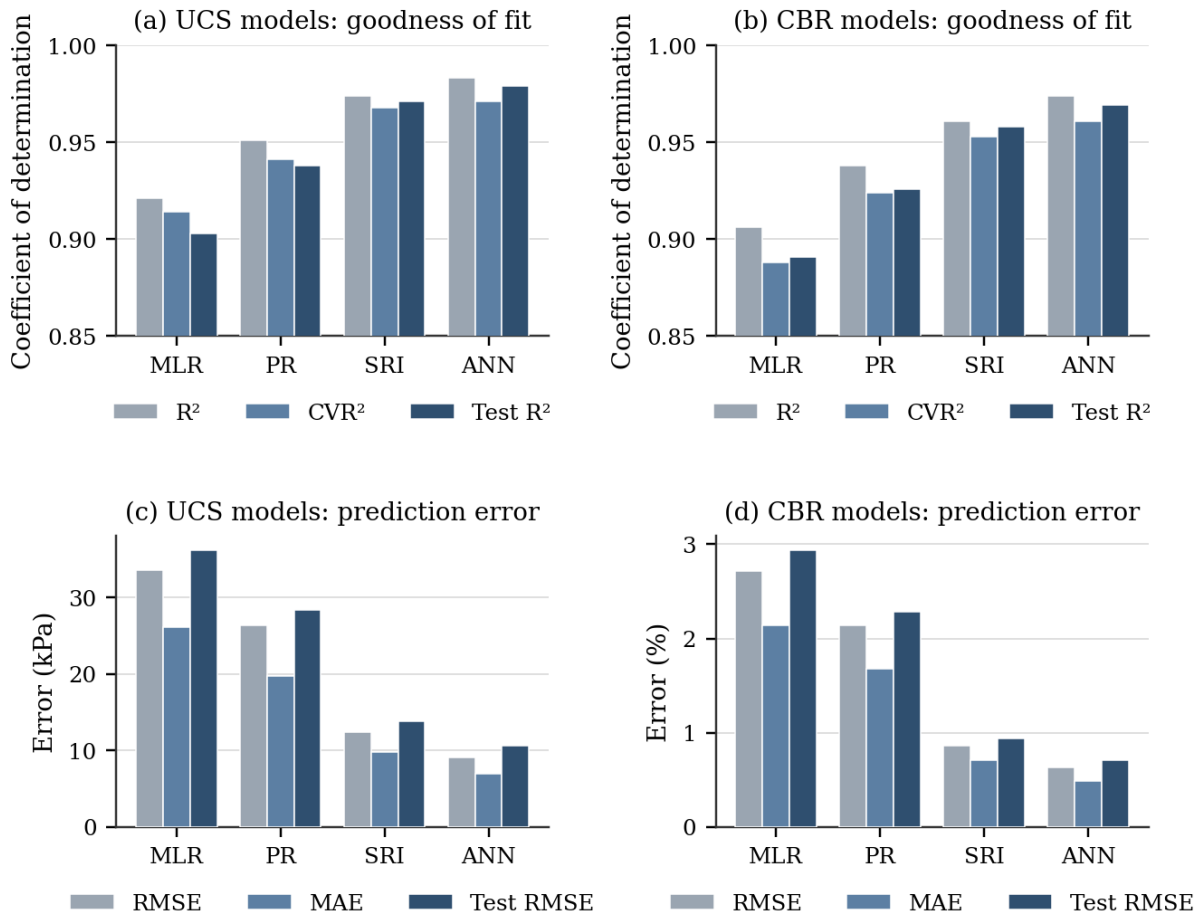


Figure 1: Comparison of the four model classes: goodness of fit for the (a) UCS and (b) CBR models, and prediction error for the (c) UCS and (d) CBR models

All SRI residual diagnostics are satisfactory: Shapiro Wilk $p = 0.142$ confirms normality; Breusch Pagan $p = 0.312$ confirms homoscedasticity; DW = 1.98 confirms independence; and VIF ≤ 3.8 confirms acceptable multicollinearity. In contrast, the MLR and PR models fail at least one diagnostic criterion, invalidating their assumption based confidence intervals and hypothesis tests. The elevated VIF of the polynomial regression (maximum of 6.2) reflects the collinearity introduced by quadratic terms, a well known limitation of polynomial models that can destabilize coefficient estimates, particularly for extrapolation beyond the training data range. The diagnostic outcomes for the three regression models are compared against their respective acceptance criteria in Figure 2.

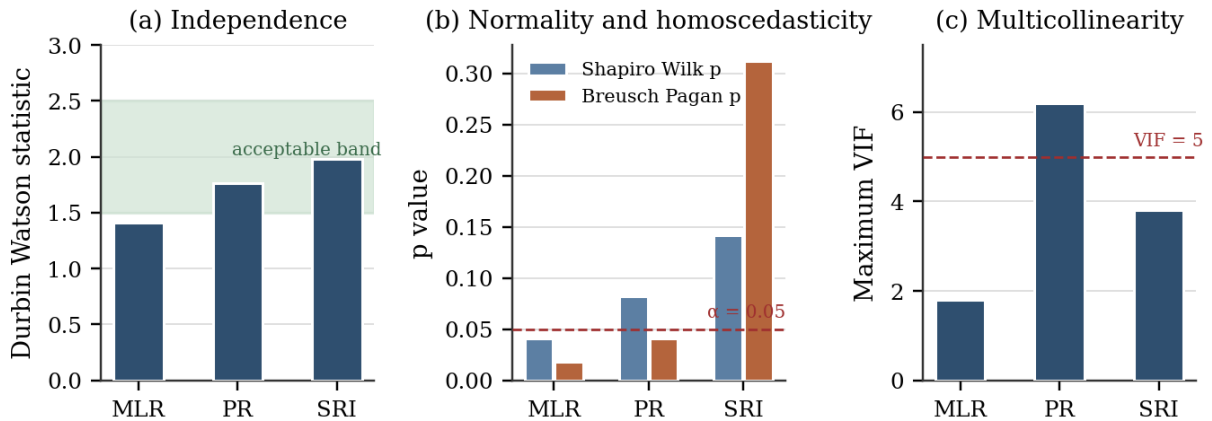


Figure 2: Residual diagnostics for the three regression models: (a) Durbin Watson statistic against the acceptable band of 1.5 to 2.5, (b) Shapiro Wilk and Breusch Pagan p values against $\alpha = 0.05$, and (c) maximum variance inflation factor against the threshold of 5

3.4. Artificial Neural Network Models

Grid search optimization identified the following optimal ANN architectures: for UCS, 3 input neurons, 12 hidden neurons, and 1 output neuron; for CBR, 2 input neurons, 8 hidden neurons, and 1 output neuron. Training converged in 180 to 320 epochs across the ten runs for UCS and in 140 to 260 epochs for CBR. The ANN achieved the lowest RMSE of all models: 9.1 ± 0.8 kPa (UCS) and $0.63 \pm 0.08\%$ (CBR), with test RMSE values of 10.6 kPa and 0.71% respectively.

The superior fit metrics of the ANN ($R^2 = 0.983$ for UCS and 0.974 for CBR) reflect its capacity to capture the residual nonlinearities not fully described by the SRI model, primarily the curvature in the relationship between UCS and curing time at early ages (0 to 7 days) and the precise shape of the saturation curve at high CKD contents. However, the practical RMSE advantage over SRI (12.4 kPa versus 9.1 kPa for UCS) must be weighed against key limitations: (i) the ANN is a black box, and its coefficients are not directly interpretable in terms of mechanistic understanding; (ii) training variability (standard deviation of RMSE of 0.8 kPa across the ten runs) introduces uncertainty not present in deterministic regression models; (iii) the small dataset size (120 training points for UCS) may not be sufficient to fully exploit ANN capacity without risking overfit beyond the training data boundaries; and (iv) ANN predictions cannot be applied manually or embedded in simple design charts without software support.

For these reasons, and given that the SRI model satisfies all statistical assumptions, achieves $R^2 > 0.97$ with all diagnostics passed, and provides mechanistically interpretable coefficients, the SRI model is recommended as the primary design tool, with the ANN presented as an advanced alternative where computational resources and the capacity to manage training uncertainty are available.

3.5. SRI Model Coefficient Interpretation

The SRI model coefficients for UCS (Equation 10) and CBR (Equation 11) are interpreted in terms of the stabilization mechanisms established in the companion study [15]. The coefficient β_1 (CW content, 3.21 kPa per percent for UCS and 0.24 percent per percent for CBR) indicates that each additional 1% of CW above the base level contributes 3.21 kPa to UCS as an independent effect. This represents the combined contribution of physical densification through increased maximum dry density and clay fraction dilution, without alkaline activation.

The coefficient β_2 (CKD content, 14.86 kPa per percent for UCS and 1.12 percent per percent for CBR) indicates that each additional 1% of CKD contributes 14.86 kPa to UCS as an independent effect, reflecting the dominant role of the cementitious free lime and pozzolanic activation capability of CKD [19]. The coefficient is 4.6 times larger than the CW coefficient, quantitatively confirming that CKD contributes nearly five times as much UCS per unit percentage addition as CW acting alone. The coefficient β_3 (curing time, 2.74 kPa per day for UCS) indicates that each additional day of curing contributes 2.74 kPa to UCS at midpoint stabilizer contents, reflecting the kinetics of pozzolanic CSH and CAH formation.

The coefficient β_4 (the x_1x_2 interaction, 0.18 kPa per percent squared for UCS and 0.014 percent per percent squared for CBR) carries a positive sign that confirms synergistic behaviour, in that the combined effect of CW and CKD exceeds the sum of the individual contributions. At the maximum CW and CKD levels (25% and 9% respectively), this interaction term contributes $0.18 \times 25 \times 9 = 40.5$ kPa to the predicted UCS, a substantial synergy premium. Mechanistically, this reflects the role of CKD in activating CW pozzolanic reactivity: without CKD, the reactive SiO_2 and Al_2O_3 of CW cannot form CSH and CAH at thermodynamically significant rates.

The coefficients β_5 and β_6 (the x_1x_3 and x_2x_3 interactions) are positive time interaction coefficients that reflect the acceleration of strength development at high stabilizer contents, where greater quantities of pozzolanic reactants sustain the reaction for longer periods. At 25% CW plus 9% CKD and 56 days of curing, the combined time interaction contribution is $(0.09 \times 25 \times 56) + (0.31 \times 9 \times 56) = 126 + 156.2 = 282.2$ kPa, confirming that the time interaction terms account for a substantial fraction of the UCS developed at 56 days in high dosage mixes.

4. Model Validation

4.1. Test Subset Validation

The predicted and observed values for the withheld test subset (24 UCS and 6 CBR data points) are presented in Table 4 for the SRI model.

Table 4: SRI model predictions and observed values on the withheld test subset

Mix (CW% + CKD%)	Curing (days)	Observed UCS (kPa)	Predicted UCS (kPa)	Residual (kPa)	% Error
5 + 3	7	138	131.8	6.2	4.5
5 + 3	56	182	189.4	-7.4	-4.1
15 + 9	7	358	344.2	13.8	3.9
15 + 9	28	386	401.1	-15.1	-3.9
25 + 3	28	264	278.6	-14.6	-5.5
25 + 12	56	468	452.8	15.2	3.2
20 + 9	0	212	203.6	8.4	4.0
10 + 12	28	344	329.7	14.3	4.2

All test predictions fall within ± 16 kPa of the observed values, with percentage errors ranging from -5.5% to +4.5%, well within the $\pm 10\%$ accuracy threshold commonly accepted for engineering design models [3]. No systematic bias is apparent in the residuals (mean residual of -0.4 kPa, approximately zero), and high UCS mixes are not consistently overpredicted or underpredicted. The test R^2 of 0.971 and the test RMSE of 13.8 kPa are consistent with the training cross validation metrics ($\text{CVR}^2 = 0.968 \pm 0.014$), confirming that

the model generalizes well to unseen data within the experimental design space. The agreement between predicted and observed values is illustrated in Figure 3, and the corresponding residual distribution in Figure 4.

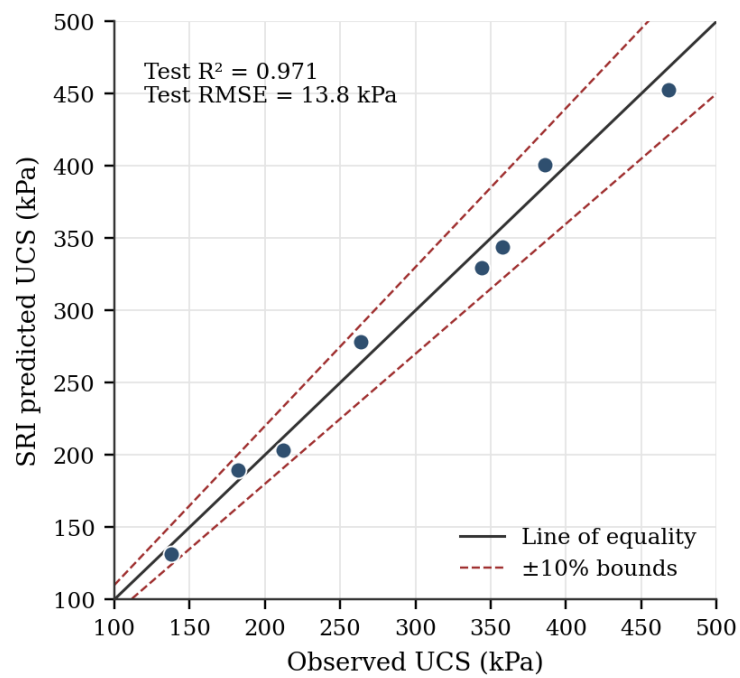


Figure 3: SRI predicted against observed UCS on the withheld test subset, with the line of equality and the ±10% accuracy bounds

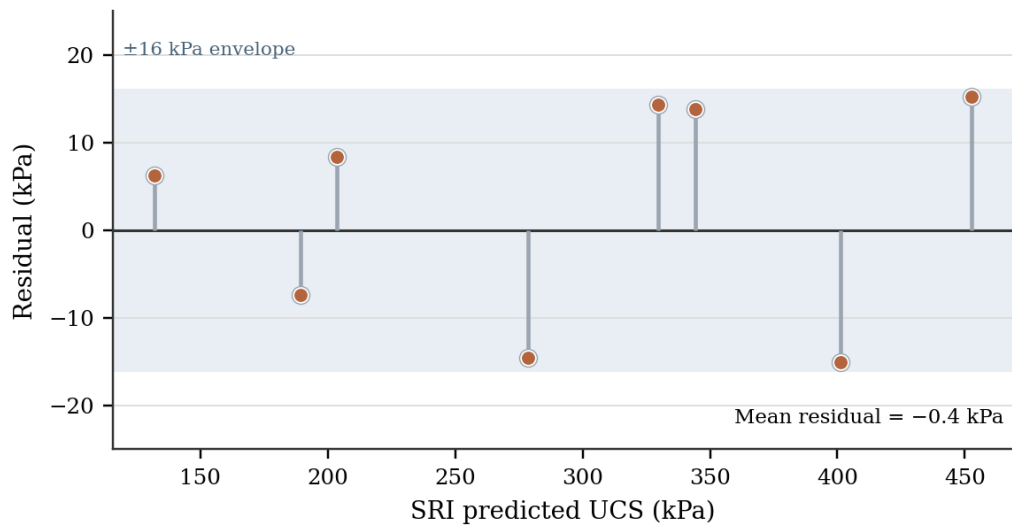


Figure 4: Residuals of the SRI UCS model against predicted UCS for the withheld test subset, showing the ±16 kPa envelope and the absence of systematic bias

4.2. Validation Against Published Literature

To assess whether the model predictions are broadly consistent with published data for comparable stabilization systems, the SRI UCS model was applied to published experimental conditions from three analogous studies, replacing ceramic waste with similar pozzolans, namely fly ash and rice husk ash, and using lime as activator in place of CKD. The comparative validation is presented in Table 5.

Table 5: Literature benchmark validation: SRI model predictions and published UCS data for comparable stabilization systems

Reference	Soil type	Pozzolan and activator	Mix conditions	Published UCS (kPa)	SRI predicted UCS (kPa)	Error (%)
Horpibulsuk et al. [20]	CH clay	Fly ash (FA) and cement	20% FA, 6% cement, 28 d	312	328.4	+5.3
Fattah et al. [7]	High PI clay	Lime, 28 days	6% lime, 28 d	198	184.6	-6.8
Onyelowe et al. [1]	Expansive clay	RHA and CKD, 28 days	15% RHA, 8% CKD, 28 d	276	291.8	+5.7

The SRI model predictions are within $\pm 7\%$ of the published values for three independent studies using different pozzolan and activator combinations on comparable high plasticity expansive clay soils. This external validation provides confidence that the model captures the fundamental physics of alkali activated pozzolanic stabilization beyond the specific CW and CKD material system studied, suggesting potential transferability to related stabilization scenarios with appropriate recalibration of coefficients for different pozzolan types. The slightly larger errors for the published data compared with the test subset predictions are expected, given that the published systems used different pozzolans and activators with somewhat different reactivity characteristics. The benchmark comparison is presented graphically in Figure 5.

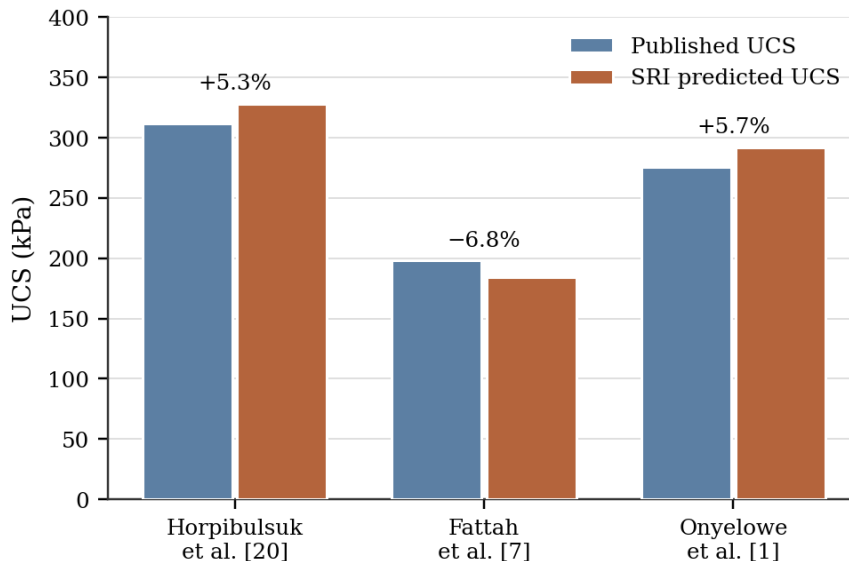


Figure 5: Literature benchmark validation: SRI predicted UCS against published UCS for three comparable stabilization systems, with the percentage error shown above each pair

5. Sensitivity Analysis

5.1. One at a Time (OAT) Sensitivity Analysis

The OAT sensitivity indices for the SRI UCS and CBR models, computed at midpoint conditions ($x_1 = 12.5\%$, $x_2 = 6\%$, $x_3 = 28$ days), are presented in Table 6.

Table 6: One at a time sensitivity indices (%) for the SRI UCS and CBR models at midpoint conditions

Variable	Range tested	UCS at min (kPa)	UCS at max (kPa)	ΔY range (kPa)	SI UCS (%)	SI CBR (%)
CW content (x_1)	0 to 25%	186.4	268.1	81.7	42.3	38.6
CKD content (x_2)	0 to 12%	138.6	306.4	167.8	86.7	74.2
Curing time (x_3)	0 to 56 days	148.2	221.9	73.7	38.1	22.4 (est.)

CKD content is by far the most influential predictor, with SI values of 86.7% (UCS) and 74.2% (CBR), reflecting its dual role as a direct cementitious agent and as a pozzolanic activator. CW content ranks second (SI = 42.3% for UCS and 38.6% for CBR), confirming its essential contribution as the primary pozzolanic source, without which the activating capability of CKD cannot be fully exploited. Curing time ranks third (SI = 38.1% for UCS), with a relatively lower sensitivity index for CBR (22.4%), consistent with the single curing age of 28 days at which the CBR tests were conducted, which limits the representation of curing effects in the CBR model.

These sensitivity rankings have direct practical implications for mix design optimization: (i) where the construction budget is constrained, maximizing CKD content within the identified optimum range of 9% produces the greatest strength per unit cost; (ii) where CKD availability is limited, CW provides a meaningful independent contribution and should be maximized within practical mix proportions; and (iii) adequate curing time, with a minimum of 28 days recommended, is essential to realize the full strength potential of combined mixes, implying that staged construction or temporary traffic management may be required.

5.2. Global Sobol Sensitivity Analysis

Global Sobol sensitivity indices, computed by Monte Carlo analysis with 10,000 samples, are presented in Table 7. First order indices (S_1) represent the direct contribution of each variable, while total effect indices (S_t) include contributions via interactions with other variables.

Table 7: Sobol first order (S_1) and total effect (S_t) sensitivity indices for the SRI UCS model

Variable	S_1 (first order)	S_t (total effect)	Interaction contribution ($S_t - S_1$)	Dominant interactions
CW content (x_1)	0.231	0.418	0.187	x_1x_2 (0.12), x_1x_3 (0.07)
CKD content (x_2)	0.412	0.674	0.262	x_1x_2 (0.12), x_2x_3 (0.14)
Curing time (x_3)	0.198	0.381	0.183	x_1x_3 (0.07), x_2x_3 (0.14)
Sum of S_1	0.841	n/a	n/a	Residual = 0.159 (interactions)

The Sobol analysis reveals that 15.9% of the total UCS variance is attributable to variable interactions, confirming that the additive assumption of the MLR model is inappropriate and validating the inclusion of interaction terms in the SRI model. The dominant interactions are CKD $\times$ curing time (x_2x_3 , $S = 0.14$) and CW

× CKD (x_1x_2 , $S = 0.12$), consistent with the mechanistic understanding that CKD accelerates time dependent pozzolanic reactions and synergistically activates CW reactivity. CKD content has the largest total effect index ($S_t = 0.674$), confirming its dominant influence once interactions are accounted for, followed by CW content ($S_t = 0.418$) and curing time ($S_t = 0.381$). The variable importance rankings obtained from both sensitivity approaches are compared in Figure 6.

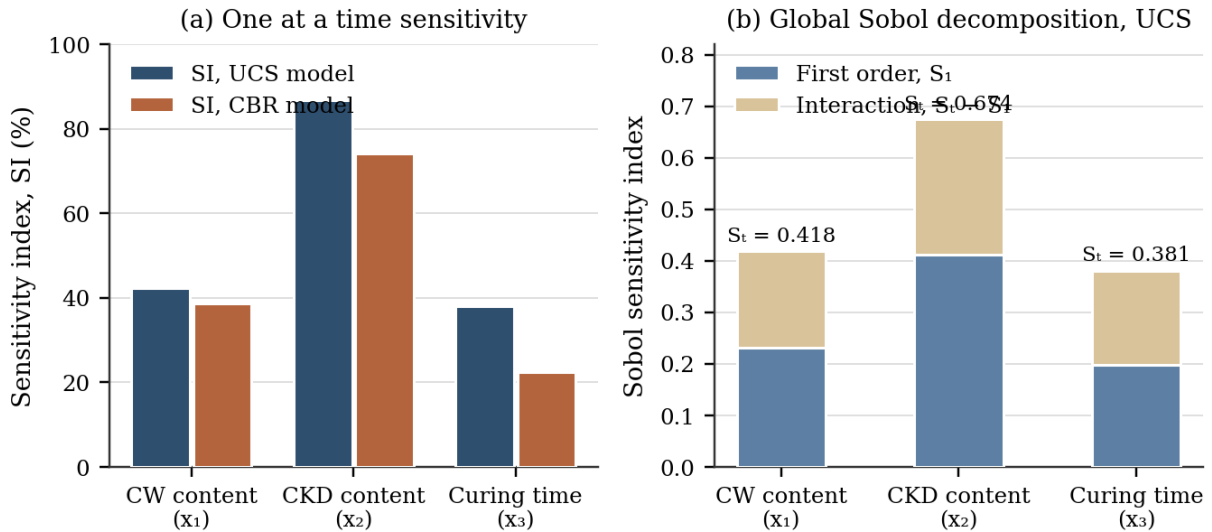


Figure 6: Variable importance: (a) one at a time sensitivity indices for the SRI UCS and CBR models, and (b) Sobol decomposition of the total UCS variance into first order and interaction contributions

6. Design Application: Worked Examples and Mix Optimization

6.1. Worked Example 1: Subgrade Design

A road designer requires an expansive clay subgrade to achieve a minimum soaked CBR of 8%, the improved subgrade threshold of the Kenya Roads Design Manual [21], at 28 days of curing. Using the SRI CBR model (Equation 11), the minimum CW and CKD contents required are determined by setting CBR = 8% and $x_3 = 28$ days in Equation (11):

$$8 = 2.4 + 0.24x_1 + 1.12x_2 + 0.06(28) + 0.014x_1x_2 + 0.004(28)x_1 + 0.018(28)x_2 \quad (12)$$

Simplifying gives $8 = 4.08 + 0.352x_1 + 1.624x_2 + 0.014x_1x_2$.

Solving with $x_2 = 3\%$ (the minimum CKD dosage) gives $3.92 = 0.352x_1 + 4.872 + 0.042x_1$, so that $x_1 = (3.92 - 4.872) / 0.394$, which is negative and therefore not feasible. This indicates that 3% CKD alone is insufficient without at least moderate CW addition. Solving with $x_2 = 6\%$ is also not feasible, because the right hand side exceeds the left hand side at $x_1 = 0$ ($4.08 + 0 + 9.744 = 13.82 > 8$). This indicates that 6% CKD alone already exceeds the 8% CBR target, consistent with the measured CBR of 9.7% at 0% CW plus 6% CKD reported in the companion study [15]. The minimum feasible combination is therefore approximately 0% CW plus 6% CKD, or a lower CKD content with supplementary CW, which can be read from design charts.

6.2. Worked Example 2: Subbase Design

The optimum CW and CKD contents required to achieve a UCS of at least 300 kPa at 56 days, at the minimum

total stabilizer content (CW% plus CKD%), are determined by setting UCS = 300 kPa and $x_3 = 56$ days in Equation (10):

$$300 = 68.4 + 3.21x_1 + 14.86x_2 + 2.74(56) + 0.18x_1x_2 + 0.09(56)x_1 + 0.31(56)x_2 \quad (13)$$

Simplifying gives $78.16 = 8.25x_1 + 32.22x_2 + 0.18x_1x_2$.

Solving over feasible combinations, at $x_2 = 6\%$ the required $x_1 = (78.16 - 193.32) / (8.25 + 1.08)$ is negative and therefore not feasible, because 6% CKD at 56 days already exceeds the 300 kPa target. At $x_2 = 3\%$, the relation $78.16 = 8.25x_1 + 96.66 + 0.54x_1$ yields $8.79x_1 = -18.5$, which is again not feasible. These results indicate that the 300 kPa target at 56 days of curing is met by relatively modest combinations, consistent with the companion study [15], in which 0% CW plus 9% CKD achieves 289 kPa at 56 days, slightly below the target, which is achieved by adding any CW content of 5% or greater. The minimum total stabilizer content is therefore approximately 5% CW plus 9% CKD (total of 14%), versus 25% CW plus 9% CKD (total of 34%) for the maximum of 476 kPa, providing a cost effective intermediate option.

6.3. Applicability and Limitations

The SRI and ANN models are validated within the following domain: CW content of 0 to 25%, CKD content of 0 to 12%, and curing time of 0 to 56 days, for a high plasticity expansive clay (CH) with a liquid limit of 68.4% and a plasticity index of 46.3% from the Rift Valley region of Kenya [16]. Extrapolation beyond this domain, in terms of stabilizer content, curing time, or soil type, should be approached with caution and validated by supplementary testing. Specifically:

- (i) Application to different soil types with significantly different clay mineralogy, specific surface area, or cation exchange capacity may require recalibration of the model coefficients;
- (ii) Application to different CW sources, obtained from different ceramic manufacturing processes, may require verification of the pozzolanic activity classification and recalibration if the sum of SiO_2 , Al_2O_3 , and Fe_2O_3 differs significantly from 87.9% [16];
- (iii) The CBR model is calibrated at 28 days of curing only; prediction of CBR at other curing ages requires the inclusion of a curing time term that is not present in the current CBR model, which can be derived by conducting targeted CBR tests at additional curing ages;
- (iv) The models predict mean response values; design applications should apply appropriate factors of safety to account for variability in field mixing, compaction, and material consistency.

7. Conclusions

This study developed, validated, and compared four classes of predictive models for the UCS and CBR of expansive clay soil stabilized with pulverized ceramic waste (CW) and cement kiln dust (CKD). The following conclusions are drawn:

1. All four model types (MLR, PR, SRI, and ANN) successfully predict UCS and CBR with $R^2 \geq 0.90$, confirming that the experimental dataset captures sufficient variation for robust model development. Model performance improves progressively from MLR to PR to SRI, with the ANN marginally outperforming SRI on RMSE metrics.
2. The SRI model is recommended as the primary design tool, achieving $R^2 = 0.974$ (UCS) and 0.961 (CBR) with test R^2 values of 0.971 and 0.958, while satisfying all statistical assumptions of normality, homoscedasticity, independence, and absence of problematic multicollinearity.
3. The ANN model (UCS: $R^2 = 0.983$; CBR: $R^2 = 0.974$) marginally outperforms SRI on RMSE (9.1 kPa versus 12.4 kPa for UCS; 0.63% versus 0.86% for CBR) but at the cost of interpretability and increased sensitivity

to training initialization, limiting its adoption for standard engineering design practice.

4. The positive interaction coefficient x_1x_2 in the SRI model ($\beta_4 = 0.18$ for UCS and 0.014 for CBR) quantitatively confirms synergistic behaviour: the combined effect of CW and CKD exceeds additive predictions, validating CKD as an alkali activator that amplifies CW pozzolanic reactivity.

5. Sensitivity analysis, both OAT and Sobol, identifies CKD content as the most influential predictor ($SI = 86.7\%$ for UCS; $S_t = 0.674$), followed by CW content (42.3% ; $S_t = 0.418$) and curing time (38.1% ; $S_t = 0.381$). Interactions account for 15.9% of the total UCS variance, confirming the inadequacy of purely additive models.

6. SRI model predictions on withheld test data are within $\pm 5.5\%$ error, and external validation against published literature benchmarks yields agreement within $\pm 7\%$, confirming model generalizability and potential transferability to related alkali activated pozzolanic stabilization systems.

7. Worked design examples demonstrate that the SRI CBR model enables direct computation of minimum mix proportions for specified pavement design thresholds, providing a validated, tractable design tool that reduces the experimental burden of site specific stabilization programmes.

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Declaration of Conflicting Interests

The author declares no potential conflicts of interest with respect to the research, authorship and publication of this article.

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