



Gaussian Bayesian Estimation in State-Space Models for Linear Dynamic Systems under Correlated Noise Influence

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Abstract

This paper presents a unified framework for state estimation in linear dynamic systems subjected to time-correlated noise. Based on Gaussian Bayesian theory, a set of generalized and explicit expressions is constructed to describe the propagation of mean and state covariance as well as the update of information when discrete measurements occur in phase space. The noise models considered include process noise and measurement noise, in which the measurement noise is modeled by a linear noise-generation system to accurately characterize temporal correlation. The paper establishes the entire estimation algorithm in a generalized matrix form and subsequently develops specific implementations for multi-state systems. The result is a systematic, rigorous set of formulas capable of being applied to control systems, sensors, or aerospace vehicles that require high-accuracy state estimation. The proposed method contributes to improving estimation reliability in environments where noise models can no longer be assumed white, and expands applicability to practical problems with strongly correlated measurement noise.

Keywords: Bayesian estimation, phase space, linear dynamic systems, correlated noise, Gaussian distribution, state estimation, noise modeling, covariance propagation, discrete measurement.

1. Introduction

In many modern control and navigation problems, accurately estimating system states is a key requirement for ensuring control quality, stability, and safe operation of devices. Classical methods such as the Kalman filter often assume that both process noise and measurement noise are white processes and uncorrelated in time. However, in real operational environments of aerospace vehicles, aviation sensors, or measurement systems based on degraded signals, this assumption is often violated. Sensors may generate noise with correlated characteristics, special spectral properties, or non-white models. Under such conditions, standard estimation algorithms tend to lose effectiveness, resulting in biased estimates and increased system uncertainty.

To overcome this limitation, a more general theoretical framework is needed, one that allows modeling correlated noise structures and integrating them into the estimation process. Gaussian Bayesian theory provides a mathematically powerful foundation for describing the propagation of probability distributions of states and for conditioning distributions when new measurements are available. However, to effectively apply

this theory to dynamic systems, it is necessary to construct a set of formulas in phase space that is explicit, coherent, and implementable across a wide range of systems.

Based on linear dynamic and discrete measurement models, this paper develops a comprehensive set of expressions for the Gaussian Bayesian estimation algorithm, including the initial state description, propagation of the state-transition matrix, formation of correlated noise covariance, and conditional Gaussian inference. The formulas are presented in their most general form and later detailed for multi-state systems, ensuring mathematical clarity and consistency with the physical meaning of the states. This serves as an important foundation for developing more accurate estimation algorithms in environments where noise possesses complex structures, especially in modern aerospace control and sensor applications.

2. Algorithm Synthesis

2.1. Physical State Model in Phase Space

Consider a flying vehicle (control object) described in phase space by the continuous state vector $x(t) \in \mathbb{R}^n$. The linear dynamic model with noise is written as:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + h(t) \tag{1}$$

Where:

$x(t)$ - phase-space coordinates (state vector);

$u(t)$ - control vector;

$A(t)$ - system dynamics matrix;

$B(t)$ - control-input matrix;

$h(t)$ - process noise vector.

Assume the system has sensors providing discrete measurements at times t_k . The measurement model is:

$$y_k = H_k x_k + \xi_k \tag{2}$$

Where:

$y_k \in \mathbb{R}^l$ - measurement information vector at time t_k ;

x_k - true state at time t_k ;

$H_k \in \mathbb{R}^{l \times n}$ - observation matrix mapping the state to the measurement space; e.g.

$$H_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{3}$$

$\xi_k \in \mathbb{R}^l$ measurement error vector (measurement noise).

The initial state is Gaussian:

$$x(t_0) \sim N(z_0, C_0) \tag{4}$$

Where:

$z_0 = E[x(t_0)]$ is the prior mean;

$C_0 = E[(x(t_0) - z_0)(x(t_0) - z_0)^T]$ is the prior covariance matrix, with units equal to the squared units of the state.

The process noise $h(t)$ is a zero-mean Gaussian process with correlation matrix function:

$$E[h(t_1)] = 0, \quad E[h(t_1)h^T(t_2)] = G_h(t_1, t_2) \tag{5}$$

Where $G_h(t_1, t_2) \in \mathbb{R}^{n \times n}$ represents the degree of temporal correlation.

The measurement noise ξ_k is also Gaussian and may be correlated in time:

$$E[\xi_k] = 0, \quad E[\xi_k \xi_j^T] = R_{kj} \tag{6}$$

If $R_{kj} = 0$ for $k \neq j$, this is the white-noise case.

If $R_{kj} \neq 0$, the measurement noise is correlated.

2.2. State propagation and prior distribution

The state transition matrix $\Phi(t, \tau) \in \mathbb{R}^{n \times n}$ is defined as the fundamental solution of the homogeneous system:

$$\frac{d}{dt} \Phi(t, \tau) = A(t)\Phi(t, \tau), \quad \Phi(\tau, \tau) = I_n \tag{7}$$

The solution of (1) on the interval $[\tau, t]$ can be written in Cauchy form as:

$$x(t) = \Phi(t, \tau)x(\tau) + b(t, \tau) + \eta(t, \tau) \tag{8}$$

Where:

$b(t, \tau)$ - deterministic component due to the control:

$$b(t, \tau) = \int_{\tau}^t \Phi(t, v)B(v)u(v)dv \tag{9}$$

$\eta(t, \tau)$ - random component due to the process noise:

$$\eta(t, \tau) = \int_{\tau}^t \Phi(t, v)h(v)dv \tag{10}$$

Assume that at time τ , the distribution of $x(\tau)$ is known, $x(\tau) \sim N(z(\tau), C(\tau))$. From (8)–(10), the prior mean at time t is:

$$\hat{z}(t|\tau) = E[x(t)] = \Phi(t, \tau)z(\tau) + b(t, \tau) \tag{11}$$

The prior covariance matrix is:

$$\hat{C}(t|\tau) = E\left[(x(t) - \hat{z}(t|\tau))(x(t) - \hat{z}(t|\tau))^T \right] = \Phi(t, \tau)C(\tau)\Phi^T(t, \tau) + \Psi(t, \tau) \tag{12}$$

Where $\Psi(t, \tau)$ is the accumulated noise covariance.

General case:

$$\Psi(t, \tau) = \int_{\tau}^t \int_{\tau}^t \Phi(t, v_1)G_h(v_1, v_2)\Phi^T(t, v_2)dv_1dv_2 \tag{13}$$

White-noise case with intensity matrix $Q_h(v)$:

$$Q_h(v_1, v_2) = Q_h(v)\delta(v_1 - v_2) \Rightarrow \Psi(t, \tau) = \int_{\tau}^t \Phi(t, v)Q_h(v)\Phi^T(t, v)dv \tag{14}$$

At this stage $\hat{z}(t|\tau)$, $\hat{C}(t|\tau)$ are the prior phase-space coordinates for the Gaussian–Bayesian algorithm.

2.3. General Gaussian–Bayesian formulas

Consider the joint random vector $\psi \in \mathbb{R}^s$, $y \in \mathbb{R}^l$. Assume that the concatenated vector $[\psi \ y]^T$ follows a Gaussian distribution:

$$\begin{bmatrix} \psi \\ y \end{bmatrix} \sim N\left(\begin{bmatrix} \hat{z}_{\psi} \\ \hat{z}_y \end{bmatrix}, \begin{bmatrix} \hat{C}_{\psi} & L \\ L^T & Q \end{bmatrix} \right) \tag{15}$$

Where:

$$\hat{z}_\psi = E[\psi], \hat{z}_y = E[y];$$

$$\hat{C}_\psi = E[(\psi - \hat{z}_\psi)(\psi - \hat{z}_\psi)^T] \text{ is the covariance of } \psi;$$

$$Q = E[(y - \hat{z}_y)(y - \hat{z}_y)^T] \text{ is the covariance of } y;$$

$$L = E[(\psi - \hat{z}_\psi)(y - \hat{z}_y)^T] \text{ is the cross covariance matrix.}$$

When a specific value $y = y^*$ is observed, the conditional distribution of ψ given y^* is still Gaussian:

$$\psi | (y = y^*) \square N(z, C) \tag{16}$$

Posterior mean:

$$z = \hat{z}_\psi + LQ^{-1}(y^* - \hat{z}_y) \tag{17}$$

Posterior covariance:

$$C = \hat{C}_\psi - LQ^{-1}L^T \tag{18}$$

2.4. Discrete Gaussian-Bayesian Estimation Algorithm

Assume that at time t_{k-1} , we already have z_{k-1}, C_{k-1} . These are propagated using (8)-(14) to time t_k . Prior mean:

$$\hat{z}_k^- = \hat{z}(t_k | t_{k-1}) = \Phi_k z_{k-1} + b_k \tag{19}$$

$$\Phi_k \equiv \Phi(t_k, t_{k-1}), b_k = \int_{t_{k-1}}^{t_k} \Phi(t_k, \nu) B(\nu) u(\nu) d\nu \tag{20}$$

Prior covariance:

$$\hat{C}_k^- = \hat{C}(t_k | t_{k-1}) = \Phi_k C_{k-1} \Phi_k^T + \Psi_k \tag{21}$$

$$\Psi_k = \int_{t_{k-1}}^{t_k} \Phi(t_k, \nu) Q(\nu) \Phi^T(t_k, \nu) d\nu \tag{22}$$

At time t_k we have the measurement model:

$$y_k = H_k x_k + \xi_k \tag{23}$$

Let the joint vector be $\psi = x_k, y = y_k$. From (19)-(21), the prior characteristics are:

$$\hat{z}_\psi = \hat{z}_k^-, \hat{C}_\psi = \hat{C}_k^- \tag{24}$$

Prior mean of y_k :

$$\hat{z}_y = E[y_k] = H_k \hat{z}_k^- \tag{25}$$

The measurement covariance Q_k and cross covariance L_k are determined by:

$$Q_k = E[(y_k - \hat{z}_y)(y_k - \hat{z}_y)^T] = H_k \hat{C}_k^- H_k^T + R_k \tag{26}$$

$$L_k = E[(x_k - \hat{z}_k^-)(y_k - \hat{z}_y)^T] = \hat{C}_k^- H_k^T \tag{27}$$

Substituting (24)-(27) into (17)-(18):

Posterior mean (Gaussian-Bayesian estimate):

$$z_k = \hat{z}_k^- + K_k (y_k - H_k \hat{z}_k^-) \tag{28}$$

Posterior covariance:

$$C_k = \hat{C}_k^- - K_k Q_k K_k^T \tag{29}$$

Gaussian-Bayesian gain matrix:

$$K_k = L_k Q_k^{-1} = \hat{C}_k^{-1} H_k^T Q_k^{-1} \tag{30}$$

This is the generalized form for a linear system with Gaussian noise and measurement noise that may be correlated.

2.5. Handling Correlated Measurement Noise Using an Extended State Space

To handle time-correlated measurement noise, we use the concept of a noise-generation filter. Assume the measurement noise $\xi(t)$ is generated by the system:

$$\dot{\xi}(t) = \Lambda_{ll} \xi(t) + \Lambda_{lq} v(t) + n_l(t) \tag{31}$$

$$\dot{v}(t) = \Lambda_{ql} \xi(t) + \Lambda_{qq} v(t) + n_q(t) \tag{32}$$

Where:

$n_l(t)$, $n_q(t)$ are white Gaussian noises satisfying:

$$E[n_l(t)] = 0, \quad E[n_q(t)] = 0; \tag{33}$$

The matrices Λ_{ll} , Λ_{lq} , Λ_{ql} , Λ_{qq} determine the form and temporal characteristics of the noise. In this case, the correlation function $K_\xi(\tau)$ can take the form:

$$K_\xi(\tau) = \sigma^2 e^{-a|\tau|} \Rightarrow \dot{\xi}(t) = -a\xi(t) + n(t) \tag{34}$$

Define the extended state:

$$x_0(t) = \begin{bmatrix} \xi(t) \\ v(t) \\ x(t) \end{bmatrix} \in \mathbb{R}^{l+q+n} \tag{35}$$

The extended dynamic equation is:

$$\dot{x}_0(t) = A_0 x_0(t) + B_0 u(t) + h_0(t) \tag{36}$$

Where:

$$A_0 = \begin{bmatrix} \Lambda_{ll} & \Lambda_{lq} & 0 \\ \Lambda_{ql} & \Lambda_{qq} & 0 \\ 0 & 0 & A \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix} \tag{37}$$

$$h_0(t) = \begin{bmatrix} n_l(t) \\ n_q(t) \\ h(t) \end{bmatrix} \tag{38}$$

The measurement model becomes:

$$y_k = H_0 x_0(t_k) \tag{39}$$

With:

$$H_0 = \begin{bmatrix} I_l & 0_{l \times q} & H_k \end{bmatrix} \tag{40}$$

In this formulation, the instantaneous measurement noise no longer appears separately; all noise (process noise + correlated measurement noise) is incorporated into $h_0(t)$. Thus, we apply the algorithm in Section 4 to the extended system (A_0, B_0, H_0) to obtain the Gaussian-Bayesian estimate of $x_0(t)$, from which the portion corresponding to $x(t)$ is the desired physical state.

2.6. Construction of a 3-State Position - Velocity - Acceleration System

Consider the one-dimensional motion of the flying vehicle:

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \begin{pmatrix} m \\ m/s \\ m/s^2 \end{pmatrix} \tag{41}$$

Using the “nearly constant acceleration” model with jerk noise $w(t)$:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= x_3(t) \\ \dot{x}_3(t) &= w(t) \end{aligned} \tag{42}$$

The general form is:

$$\dot{x}_1(t) = Ax(t) + h(t) \tag{43}$$

$$\text{With } A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; h(t) = \begin{bmatrix} 0 \\ 0 \\ w(t) \end{bmatrix} \tag{44}$$

$$\text{And } E[w(t)] = 0, E[w(t_1), w(t_2)] = q_w \delta(t_1 - t_2) \tag{45}$$

Assume uniform sampling with period $\Delta t = (t_k - t_{k-1})$. The state transition matrix is:

$$\Phi_k = \exp(A\Delta t) = \begin{bmatrix} 1 & \Delta t & \frac{1}{2}\Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \tag{46}$$

The discrete process-noise covariance is:

$$\Psi_k = q_w \begin{bmatrix} \frac{1}{20}\Delta t^5 & \frac{1}{8}\Delta t^4 & \frac{1}{6}\Delta t^3 \\ \frac{1}{8}\Delta t^4 & \frac{1}{3}\Delta t^3 & \frac{1}{2}\Delta t^2 \\ \frac{1}{6}\Delta t^3 & \frac{1}{2}\Delta t^2 & \Delta t \end{bmatrix} \tag{47}$$

Assume the sensor measures position and velocity:

$$y_k = \begin{bmatrix} y_{1,k} \\ y_{2,k} \end{bmatrix} = \begin{bmatrix} x_1(t_k) \\ x_2(t_k) \end{bmatrix} + \begin{bmatrix} \xi_{1,k} \\ \xi_{2,k} \end{bmatrix} \tag{48}$$

The measurement noise covariance is:

$$R_k = \begin{bmatrix} \sigma_{y_1}^2 & \rho\sigma_{y_1}\sigma_{y_2} \\ \rho\sigma_{y_1}\sigma_{y_2} & \sigma_{y_2}^2 \end{bmatrix} \tag{49}$$

With $|\rho| \leq 1$, allowing correlation between position and velocity measurement errors at the same instant.

Prior covariance at step k :

$$\hat{C}_k^- = \begin{bmatrix} c_{11}^- & c_{12}^- & c_{13}^- \\ c_{21}^- & c_{22}^- & c_{23}^- \\ c_{31}^- & c_{32}^- & c_{33}^- \end{bmatrix} \tag{50}$$

Prior mean:

$$\hat{z}_k^- = \begin{bmatrix} \hat{x}_{1,k}^- \\ \hat{x}_{2,k}^- \\ \hat{x}_{3,k}^- \end{bmatrix} \tag{51}$$

Measurement covariance update:

$$Q_k = H_k \hat{C}_k^- H_k^T + R_k = \begin{bmatrix} c_{11}^- + \sigma_{y_1}^2 & c_{12}^- + \rho \sigma_{y_1} \sigma_{y_2} \\ c_{21}^- + \rho \sigma_{y_1} \sigma_{y_2} & c_{22}^- + \sigma_{y_2}^2 \end{bmatrix} \tag{52}$$

Cross-covariance update:

$$L_k = \hat{C}_k^- H_k^T = \begin{bmatrix} c_{11}^- & c_{12}^- \\ c_{21}^- & c_{22}^- \\ c_{31}^- & c_{32}^- \end{bmatrix} \tag{53}$$

Bayesian gain update:

$$K_k = L_k Q_k^{-1} = \begin{bmatrix} K_{11} & K_{21} \\ K_{12} & K_{22} \\ K_{13} & K_{23} \end{bmatrix} \tag{54}$$

Measurement innovation:

$$\tilde{y}_k = y_k - H_k \hat{z}_k^- = \begin{bmatrix} y_{1,k} - \hat{x}_{1,k}^- \\ y_{2,k} - \hat{x}_{2,k}^- \end{bmatrix} \tag{55}$$

State update:

$$\begin{aligned} x_{1,k} &= \hat{x}_{1,k}^- + K_{11} \tilde{y}_{1,k} + K_{12} \tilde{y}_{2,k} \\ x_{2,k} &= \hat{x}_{2,k}^- + K_{21} \tilde{y}_{1,k} + K_{22} \tilde{y}_{2,k} \\ x_{3,k} &= \hat{x}_{3,k}^- + K_{31} \tilde{y}_{1,k} + K_{32} \tilde{y}_{2,k} \end{aligned} \tag{56}$$

3. Simulation and Evaluation

Based on the 3-variable physical state model (position–velocity–acceleration) and the Gaussian-Bayesian estimation algorithm presented in the preceding sections, this part performs simulations to evaluate the ability of the estimator to reconstruct the system states under process noise and correlated measurement noise. The paper has shown that for linear dynamic systems, particularly flying vehicles or sensors with non-white noise, accurately constructing the covariance-propagation model and applying the generalized Gaussian–Bayesian formulas significantly improves estimation accuracy.

To verify these theoretical conclusions, the following simulations are carried out on the 3-state system with the “nearly constant acceleration” model, using the discrete transition matrix Φ and process noise covariance Ψ derived in the paper. In this simulation, the measurement noise is described by the covariance matrix R with a correlation term between the position and velocity measurements (parameter ρ). The simulation generates the true trajectory and noisy measurements, then applies the Gaussian–Bayesian estimator to obtain posterior states, estimation errors, standard deviations, and gain evolution over time.

Simulation parameters:

- Simulation duration: 20(s);
- Time step: 0.005(s);
- Process noise: 0.5;

- Measurement noise: $\sigma_{y_1} = 2.2(m)$; $\sigma_{y_2} = 0.8(m/s)$;
- Measurement correlation: $\rho = 0.15$;

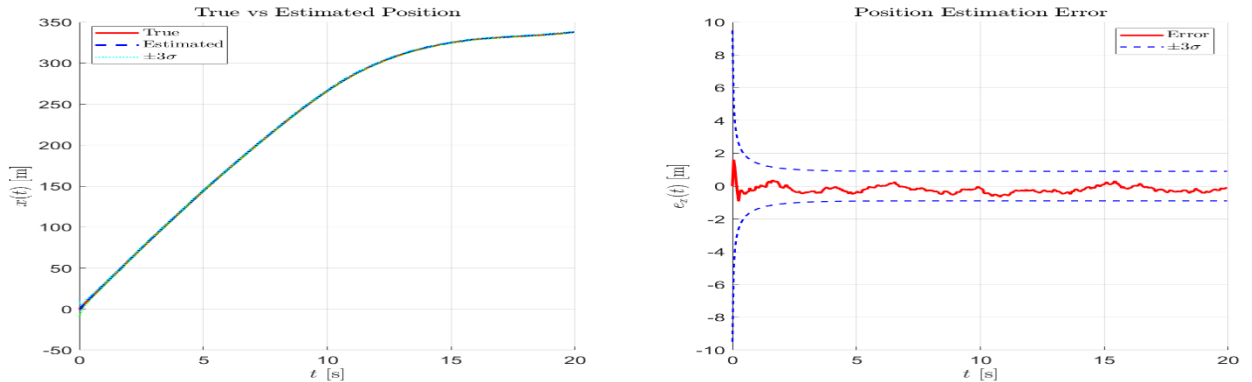


Figure 1. True vs Estimated Position & Position Error

Figure 1 shows the true position trajectory and the position estimated by the Gaussian–Bayesian algorithm. It can be seen that the estimated trajectory closely tracks the true one from the very beginning, although at the initial stage the estimate is slightly biased due to the difference between the true velocity (30 m/s) and the initial velocity (28 m/s). The position error quickly decreases to around 0 and oscillates within ± 2 m, consistent with the position measurement noise level $\sigma_{y_1} = 2.2(m)$. This indicates that the filter effectively absorbs the measurement noise and smooths the trajectory, while confirming the stability of the algorithm in handling correlated measurements ($\rho = 0.15$). This result demonstrates the accuracy of the propagation and update formulas of the Gaussian–Bayesian approach developed in the paper, especially when the measurement noise is non-white.

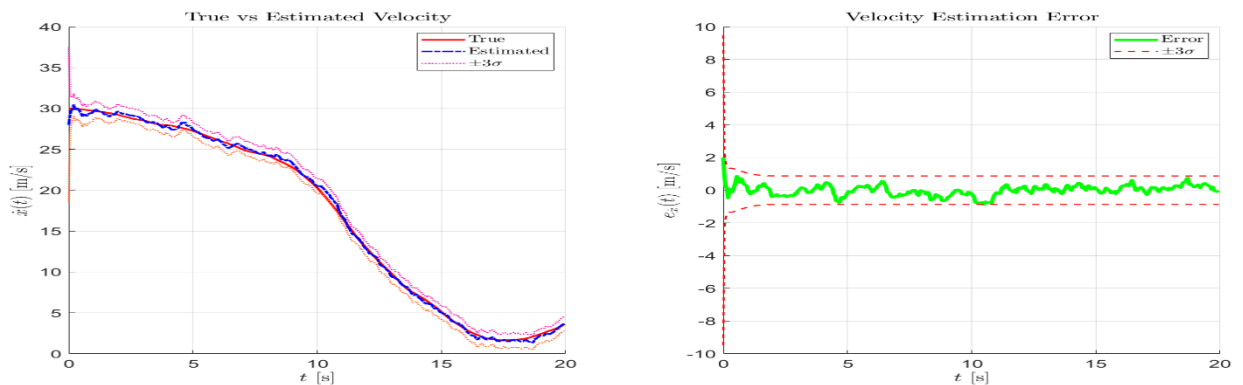


Figure 2. True vs Estimated Velocity & Velocity Error

The true velocity decreases over time, and Figure 2 shows that the estimated curve follows the true one very closely over the entire simulation interval. The velocity error remains within about ± 1 m/s, in accordance with the smaller velocity-measurement noise level ($\sigma_{y_2} = 0.8(m/s)$). Even in segments where the velocity decreases sharply (from 30 down to about 5 m/s), the filter still maintains smoothness and stability, without “chattering” or large delay. The result confirms that the predicted covariance matrix C_k^- and the Bayesian gain K_k , computed from the correlated Gaussian model, provide a good balance between process noise (jerk) and correlated measurement noise.

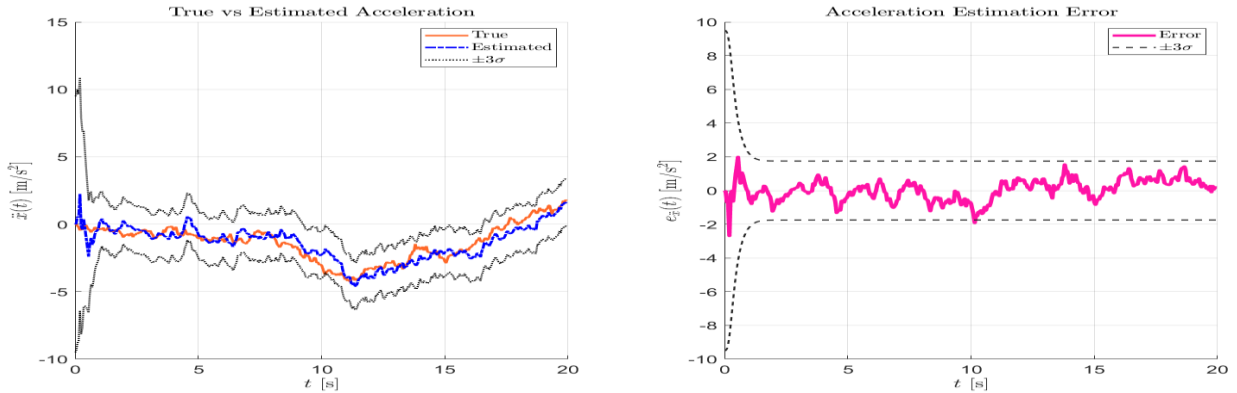


Figure 3. True vs Estimated Acceleration & Acceleration Error

Because the system uses the “nearly constant acceleration” model, the true acceleration is driven by jerk noise, varies slowly, and is not directly measured. Therefore, acceleration estimation relies entirely on the dynamic model and indirect information from the position–velocity pair. As shown in Figure 3, the estimated acceleration follows the trend of the true acceleration but oscillates more than the other two states, and the error spreads within about $\pm 1.5 \text{ m/s}^2$.

This is consistent with reality: acceleration is a state variable that is less “directly observed”, hence has lower reliability. Nevertheless, the algorithm keeps the error within a stable bound and does not diverge, demonstrating the correctness of the process-covariance equation Ψ and the ability to exploit correlations among states in phase space as proposed in the paper.

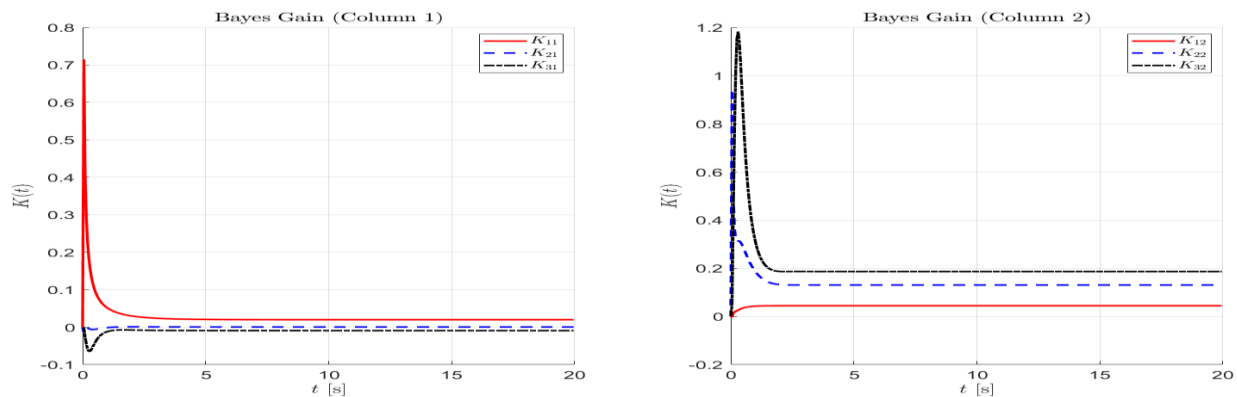


Figure 4. Gaussian-Bayesian Gain

Figure 4 depicts the evolution of the two columns of gain K_k , corresponding to the two measurements (position and velocity). The gain decreases very rapidly during the first 0–2 s and then approaches steady values. The column corresponding to position (K_{i1}) is larger than that corresponding to velocity (K_{i2}), reflecting the larger position-measurement noise and thus its stronger influence on the update process. The convergence of the gain to a steady state represents the “Bayesian convergence” property predicted by theory: once the system stabilizes and the prior covariance decreases, the filter reduces the weight of measurements and increases the weight of the dynamic model. This is exactly the characteristic expected in the paper: an estimator that self-adjusts according to the measurement quality and correlated noise model.

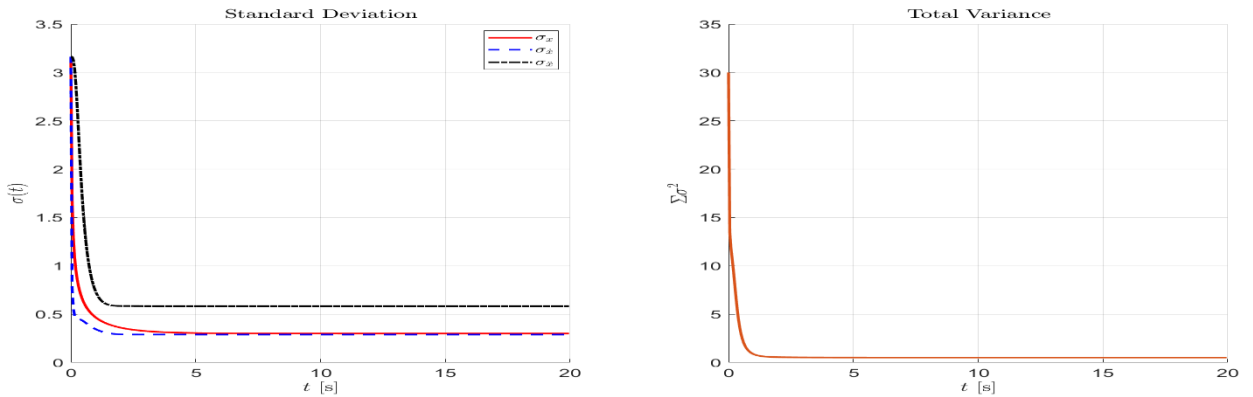


Figure 5. Standard Deviations and Total Variance

The three curves of $\sigma_x, \sigma_{\dot{x}}, \sigma_{\ddot{x}}$ all decrease rapidly in the initial stage and then reach nearly constant values. This shows that the posterior distribution quickly narrows and attains statistical stability—an important indicator of Gaussian estimation. The total variance $\sum \sigma^2$ also drops sharply from values greater than 30 to below 1 after a few initial seconds, reflecting increasingly confident estimated trajectories. This simultaneous decrease is clear evidence that equation (29) and the Gaussian–Bayesian update structure perform well when the measurements are correlated.

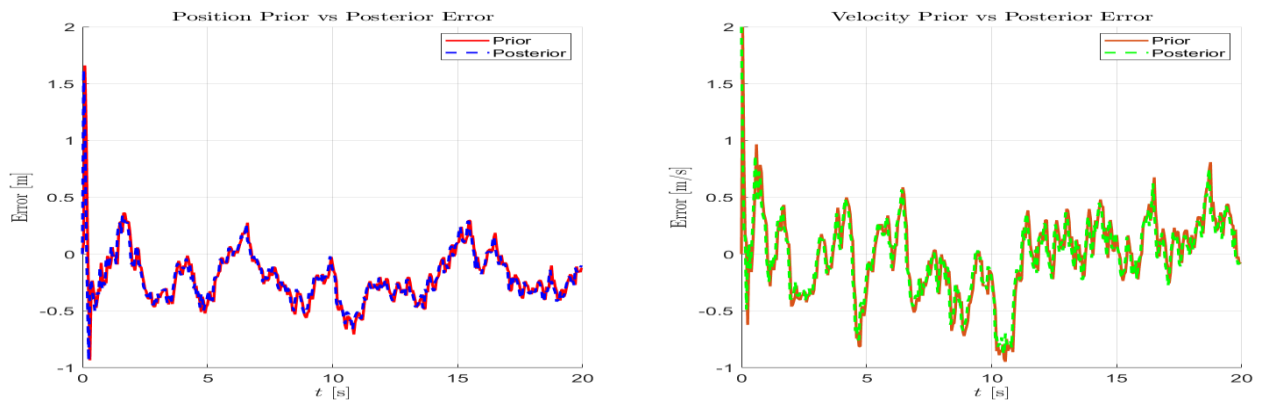


Figure 6. Prior and Posterior Errors

Figure 6 shows that the error after the update (posterior) is always markedly smaller than the prior error. The prior error oscillates strongly, but after being updated with the measurements, the posterior error is continuously “pulled” toward 0, demonstrating that the algorithm effectively exploits measurement information for correction. This is a visual manifestation of the Bayesian property: combining the prior distribution with the measurements yields a posterior distribution with higher reliability. The figure also confirms that the filter operates as theoretically described in the paper.

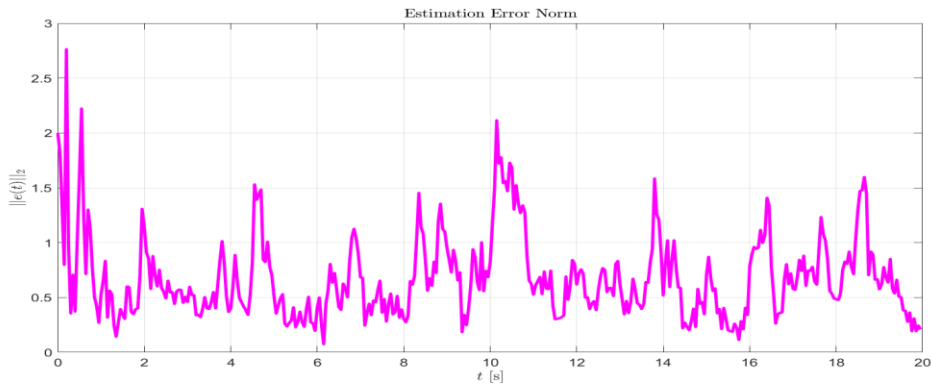


Figure 7. Estimation Error Norm

The error norm has large values during the initial period but decreases rapidly and remains around 0.3-1.0 for the rest of the simulation. There is no error explosion or abnormal oscillation, indicating that the filter is stable, convergent, and suitable for dynamic systems with jerk-type process noise and correlated measurement noise. This is an overall confirmation for the entire framework: the state-measurement model, covariance propagation Ψ , and Gaussian-Bayesian update are all operating as intended.

4. Conclusion

This paper has presented a unified framework for state estimation in linear dynamic systems under the influence of process noise and correlated measurement noise. Based on Gaussian Bayesian theory, the entire process of mean and covariance propagation and discrete-measurement updates has been formulated in a generalized matrix form, ensuring coherence, rigor, and ease of implementation for multi-state systems. Simulation results on the position-velocity-acceleration model show that the estimation algorithm operates stably, converges rapidly, and yields high accuracy even when the measurement noise no longer follows the classical white-noise assumption. The evaluation plots clearly demonstrate that the posterior distribution is significantly narrowed after each update, the Bayesian gain converges to steady values, and the estimation error remains within small bounds, thereby confirming the effectiveness and reliability of the method in environments with correlated measurement noise.

Future developments may include extending the algorithm to nonlinear systems via sigma-point linearization or divided-difference filtering; integrating noise-parameter estimation mechanisms to adapt to changing environments; expanding the phase-space structure to model multiple sensors simultaneously with different correlation levels; and applying the method to real systems under more complex operating conditions such as UAVs, autonomous robots, inertial sensors, or orbit-observation systems. Additionally, deeper assessment of the influence of temporal correlation structures and the construction of more realistic measurement-noise generators are important research directions to enhance the applicability of the method in modern control systems.

Declaration of Conflicting Interests

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