International Journal of Science, Architecture, Technology, and Environment Volume 03, Issue 04, April 2026
ISSN 3048-8222 (Online) | www.ijsate.com | editor@ijsate.com

International Journal of
Science, Architecture, Technology and Environment

Problem Formulation on Optimum Production Capacity to Optimize in Linear Programming Technique

Anil Raj[1*], Dr. Krishnandan Prasad[2]

[1]Research Scholar, Department of Mathematics, Patliputra University, Patna

[2]Professor & Head, Department of Mathematics, T.P.S. College Patna, PPU. Email Id-anilraj470@gmail.com

*Corresponding author

DOI: https://doi.org/10.63680/ijsate0426187.125

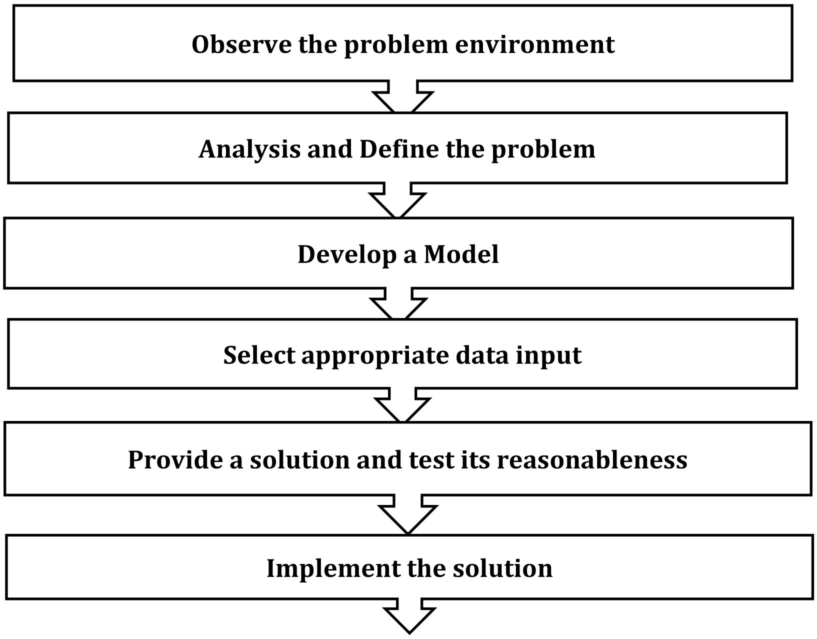
Abstract

In this Paper, we have concerned with applying linear programming technique to problem formulate optimum production. Many optimum production capacity problems involve in our life conflicting goal achieve by linear programming technique. We give an up-to-date overview of how important ideas from optimization and multi-criteria decision analysis are blended to situation where the existence of several objective function. Given Example, we formulate an optimum production capacity in a factory production cost in Linear Programming Technique.

Keywords: Optimum Production, Linear Programming Technique.

1. Introduction

Linear Programming is a discipline that provided scientific method for the purpose of solving real life problems that help us in determining the best utilization of limited resource. Linear Programming came in to existence and discovered during the World War II in Britain with the establishment of team of scientists. Scientists of different disciplines were part of this team, research on military operation soon find application in other fields also. Here we study about linear programming technique in Linear Programming. In our daily life, we observe many situations of Linear Programming around us. As example we want to maximize the profit or minimize the cost then maximization of the profit or minimization of cost is the optimize cost. Now, it was started applying the fields of industry, trade, agriculture planning and various other fields of economy. A mathematical programming is an optimization technique by which the maximum or minimum value of a function is determined under certain condition. Mathematical programming in which constraints are expressed as linear equalities.



Linear Programming a Mathematical technique for generating and selecting the optimal or the best solution for a given objective function. It may be defined as method of optimizing (Maximizing or Minimizing) a linear function for a number of constraints in the form of linear equation. Many problems from industry alive a combinatorial natural.

Problem Definition and Mathematical Formulation

An example of the first problem to be solved as a test of the simplex method. The problem is to a factory produce at minimum cost at given the price and content of each allowable. The formulation of this linear programming has been modified for optimum production capacity.

2.1 The mathematical Model:

The mathematical model of a linear programming problem is presented as:

Find C= (c_1;c_2;……c_p) so as to,

$$max: Z = \sum_{j=1}^{p} y_j c_j \quad (i)$$

Subject to:

$$\sum_{j=1}^{p} u_{ij} c_j \leq \{v_{i1}, v_{i2}, \dots \dots v_{ix_i}\}, i = 1,2 \dots \dots q \qquad (ii)$$

$$c_j \geq 0, \qquad j = 1,2 \dots \dots \dots p \quad (iii)$$

Each constraint *(ii)* has a set of x_i number of achieves where only one achieve is to be selected to solve the linear programming problem *(i)-(iii)* it is necessary to transform the problem to a standard mathematical programming problem. A general transformation is presented for all cases. Our technical works not only for small values of x_i but also for the large values.

2.2 Linear Transformation Technique:

Let $v_{i1}, v_{i2}, \dots \dots v_{ix_i}$ denote the set of values in the right hand side of the t^{th} constrain with total number x_i

K: =Least common multiple of the numbers $\{x_1, x_2, \dots \dots x_q\}$ *(iv)*

In order to achieve our main aim of selecting just one value among the x_i choice, our idea depend on inserting a set of K binary variables, our idea depend on inserting a set of K binary variables, namely: $z_1, z_2, \dots \dots z_k$ to the construct a set of $s_i = \frac{k}{x}$ linear combinations in the following form:

$$M_{i1} = h_{i1}z_1 + k_{i2}z_2 + \dots \dots \dots + h_{ix_i}z_{x_i} = \sum_{t=1}^{x_i} h_{it}z_t$$
$$M_{i2} = h_{i2}z_{x_{i}+1} + h_{i2}z_{x_{i}+2} + \dots . + h_{ix_i}z_{2x_i} = \sum_{t=1}^{x_i} h_{it}z_{x^{+t}}$$
$$M_{i3} = h_{i1}z_{2x_{i}+1} + h_{i2}z_{2x_{i}+2} + \dots \dots \dots + h_{ix_i}z_{3x_i} = \sum_{t=1}^{x_i} h_{it}\, z_{2x_{i}+t} \quad (v)$$
$$\vdots \qquad \qquad \vdots$$
$$M_i, s_i = h_{i1}\, z_{(s_{i-1})x_{i}+1} + h_{i2}z_{(s_{i}-1)x_{i}+2} + \dots \dots \dots + h_{ix_i}z_{s_ix_i} = \sum_{t=1}^{x_i} h_{i,t}\, z_{(s_i-1)x_{i}+t}$$

Finally, we replace the right of i^{th} constrain.

$$M_i = \sum_{j=1}^{s_i} M_{ij} = \sum_{j=1}^{s_i}\left\{\sum_{t=1}^{x_i} h_i{}^t\, z_{(j-1)x_{i}+t}\right\} \quad (vi)$$

Now, we can rewrite the mathematical model *(i)-(iii)* in more convenient form as:

Find c = $(c_1, c_2, c_3 \dots \dots . c_n)$ so as to:

$$\text{Max } z = \sum_{j=1}^{p} y_j c_j \quad (vii)$$

Subject to:

$$\sum_{j=1}^{p} u_{ij}\, c_j \le L_i, i = 1,2 \dots \dots . q \quad (viii)$$
$$z_\omega = 0\backslash 1, \omega = 1,2 \dots \dots \dots . k \quad (ix)$$
$$\sum_{\omega=1}^{k} z_p = 1, c_j \ge 0, j = 1,2 \dots \dots . p \quad (x)$$

Where M_i, given *(vi)*. The restriction $\sum_{\omega=1}^{k} z_w = 1$ mentioned in *(x)* implies that one and only of the k's binary variable will take values one while the remaining *(k-1)*'s will take the value zero which ensures the achievement of our main aim of selecting a unique parameter value among the x_i's values in the each constrain.

2.3 General Linear Programming Problem:

Let us formulate the general programming problem.

Let z be a linear function of p basic variables $c_1, c_2, c_3 \dots \dots . c_n$

Which is to be Maximize (or Minimize)

$$Z = y_1c_1 + y_2c_2 + y_3c_3 + \dots \dots \dots + y_pc_p \quad (xi)$$

Where $y_1, y_2, y_3 \dots \dots \dots$ ..are known constant termed as cost coefficient of basis variables.

Let (u_{ij}) be a $p \times q$ real matrix of $p \times q$ constants u_{ij} and let $\{h_1, h_2, \dots \dots . h_q\}$ be a set of constants such that

$$u_{11}c_1 + u_{12}c_2 + \dots \dots \dots \dots + u_{1p}c_p \le or = or, \ge h_1$$
$$u_{21}c_1 + u_{22}c_2 + \dots \dots \dots \dots + u_{2p}c_p \le or = or, \ge h_2 \quad (xii)$$
$$\vdots \qquad \vdots$$
$$u_{q1}c_1 + u_{q2}c_2 + \dots \dots \dots \dots u_{pq}c_p \le or = or, \ge h_q$$
$$\text{And } c_j \ge 0 \text{ for all } j = 1, 2, 3 \dots \dots \dots p \quad (xiii)$$

The linear function z in equation *(xi)* is called the objective function. The set of inequalities given in *(xii)* is called constraints of a general and the set of inequalities given in *(xiii)* are known as non-negative restrictions of a general LP.

Model	Royal a_1	Royal Max a_2	Royal Ultra a_3	Total avail hours
Labour	10 hours	4	5	900 or 1000 or 1100
Technical Time	5 hours	7	2	1000 or 1450 or 1600 or 2000
Packing Hours	1 hour	2	3	300 or 400
Profit	$130	$100	$80	

Table 1.

3. Numerical Example:

Case (i)

A factory produces three different types of Air Condition (AC): Royal, Royal Max, and Royal Ultra. Production of one AC of type Royal requires 10 hours general labour, 5 technical hours and 1 packing hour. While the AC of type Royal Max requires 4 hours general labour, 7 technical and 2 packing hours and the AC type of Royal Ultra 5 general, 2 technical hours and 3 packing hours. The factory can afford up to 900 or 1000 or 1100 hours of general labour hours, up to 1000 or 1450 or 1600 or 2000 hours of technical labour and up to 300 or 400 packing hours per week. AC of type Royal, type Royal Max and Royal ultra earns a profit of $ 130, $100 and $80 respectively. The factory management decided to produce at least 20 AC per week should the factory produce from each type to Maximum profit.

Solution: Let $a_1, a_2, and\ a_3$ denote the number of produced AC of type Royal, Royal Max and Royal Ultra.

The following table summarize the informative above: ***Table 1.***
Since, the production manager wants to maximize the profit.
so, we have following mathematical problem.

$$Max\ z = 130a_1 + 10a_2 + 80a_3 \quad (xiv)$$

Subject to the Constraints:

$$10a + 4a_2 + 5a_3 \leq \{900,1000,1100\} \quad (xv)$$
$$5a_1 + 7a_2 + 2c_3 \leq \{1000,1450,1600,2000\}$$
$$a_1 + 2a_2 + 3a_3 \leq \{300,400\} \quad (xvi)$$
$$a_1, a_2 \geq 0, a_3 \geq 20 \quad (xvii)$$

Applying the above technique in (v) and (vi), we can transform the given programming problem to a mixed integer programming problem.
We have,

$$x_1 = 3, x_2 = 4, x_3 = 2.$$

So, *k=12* which denotes the least common integer *(3, 4, 2)*
Therefore, we insert 12 binary variable in the right side of the constraints.

$$\text{Max } z = 130a_1 + 100a_2 + 80a_3 \quad (xviii)$$

Subject to:

$$10a_1 + 4a_2 + 5a_3 + \leq 900z_1 + 1000z_2 + 1100z_3 + 900z_4 + 1000z_3 + 1100z_6 \quad (xix)$$
$$5c_1 + 7a_2 + 2c_3 \leq 1000z_1 + 1450z_1 + 1600z_3 + 2000z_4 + 1000z_5 + 1450z_6 + 1600z_7 + 2000z_8 + 1000z_9 + 1450z_{10} + 1600z_{11} + 2000z_{12} \quad (xx)$$

$$a_1 + 2a_2 + 3a_3 \leq 300z_1 + 400z_2 + 300z_3 + 400z_4 + 300z_5 + 400z_6 + 300z_7 + 400z_8 + 300z_9 + 400z_{10} + 300z_{11} + 400z_{12}$$

$$z_\omega = 0\backslash 1, \omega = 1,2 \ldots \ldots \ldots .12 \quad \textit{(xxi)}$$

$$\sum_{\omega=1}^{12} z_w = 1, a_1, a_2 \geq 0, a_3 \geq 20 \quad \textit{(xxii)}$$

Case (ii)

A small scale bread manufacturing industry making two product Simple Bread and Brown Bread which are processed in two step: first step Spiral Mixer, second step is Bread Tunnel Over. Simple bread requires 2 hours of work in spiral mixer and 4 hours of work in bread tunnel oven to making, while brown bread requires 3 hours of work in the spiral mixer and 2 hours of work in the bread tunnel oven. In one day, the industry cannot use more than 16 hours of spiral mixer and 22 hours of bread tunnel oven. The manufacturing industry makes a profit of rupees 3 per unit of product while simple bread, 4 per unit of product brown bread.

Solution: Let a and b be the number of units of product simple bread and brown bread, which are to be produced

Here *a* and *b* are the decision variable.

Since, one unit of product Simple Bread and one unit of product Brown Bread gives the profit of ₹3 and ₹4, respectively the objective function is:

Maximize z= 3a+4b

The requirement and availability in hours of each of the machine for making the product in Mention Table 2, &3.

Type of Bread	Spiral Mixer	Bread Tunnel Oven
Simple Bread	2 hours	4 hours
Brown Bread	3 hours	2 hours
Working Hours		

Table 2.

Type of Bread	Profit ₹ (per unit)
Simple Bread	3
Brown Bread	4

Table 3.

Total hours of spiral mixer required for both Type of product = 2a+3b

Total hours of Bread Tunnel Over required for both Type of product = 4a+2b

Hence, the constraints as per the limited available resource are: 2a+3b⩽16 and 4a+3b⩽22

Since, the number of units produced for both Simple Bread and Brown Bread cannot be negative, a⩾0 ,b⩾0

Thus, the mathematical formulation of the given problem is maximizing z = 3a+4b

Subject to the constraints:

2a+3b⩽16

$4a+3b \leqslant 22$

And non-negative restrictions

$a \geqslant 0, b \geqslant 0$

After formulating a linear programming problem, to solve it, we learnt that linear programming problem can be represented as problems of maximisation or minimisation with constraints such as ⩽,=,⩾. In order to develop a standard procedure for solving LPP.

Conclusion & Result:

The aim of this article has to describe a new efficient technique for solving mathematical formulation problems. The process depends on obtaining a number of linear mathematical expression equal to the number of constrains and contain a specified number of binary variables we tried to set the models as general as possible and that way make then application to any given mathematical formulation of linear programming. We can describe that these models can serve as a simple and easy method for solving this type of mathematical problems using available resource. Linear problem is considered main advantage of the method.

Declaration of Conflicting Interests

The authors declare no potential conflicts of interest with respect to the research, authorship and publication of this article.

Funding

The author received no financial support for the research, authorship and publication of this article.

References

1. Dantzig, G.B. (1963) Linear Programming and Extensions. Princeton University Press, Princeton.
2. Hiller, F. and Lieberman, G. (1990) Introduction to Operations Research. McGraw-Hill, New York.
3. Chary, S. N., Production and Operation Management, Tata McGraw-Hill Publishing Company Limited, Second edition 2000.
4. Vaidya SO, Kumar S. Analytic hierarchy process: An overview of applications. European Journal of Operational Research 169 (2006) 1–29 2006; 169: 1-29.
5. Wagner, H.M. (1969) Principles of Operations Research. Prentice-Hall Inc., Engle wood Cliffs, New Jersey.
6. Frizzone, J.A., Coelho, R.D., Dourado-Neto, D. and Soliani, R. (1997) Linear Programming Model to Optimize the Water Resource Use in Irrigation Projects: An Application to the Senator Nilo Coelho Project. Scientia Agricola, 54, 136-148.
7. Akpan, N. and Iwok, I.A. (2016) Application of Linear Programming for Optimal Use of Raw Materials in Bakery. International Journal of Mathematics and Statistics Invention, 4, 51-57.