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Performance Evaluation of Multiple Tuned Mass Dampers for Seismic Vibration Control of Multi-Storey Structures

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Abstract

This study evaluates the seismic response of structural systems subjected to base excitation using tuned mass dampers (TMDs). The performance of Multiple Tuned Mass Dampers (MTMDs) is compared with that of a single TMD. Linear time-history analysis is performed on a plane frame model under strong ground motions from four well-known earthquakes with varying peak ground accelerations. Structural responses are examined for configurations with and without dampers. The influence of key design parameters—mass ratio, damping ratio, tuning frequency ratio, frequency spacing, and number of dampers—is investigated. Results demonstrate that optimally designed MTMD systems improve vibration control performance and provide enhanced robustness compared with a single TMD system.

Keywords: Seismic response, Tuned mass damper, MTMD, Time-history analysis, Vibration control

1. Introduction

A tuned mass damper (TMD) is a passive vibration control device consisting of a secondary mass connected to the primary structure through a spring and viscous damper. The device reduces structural vibration by transferring energy from the main system to the auxiliary mass, where it is dissipated through damping. Effective performance requires tuning the damper frequency close to the natural frequency of the structure.

The concept of vibration absorption was first introduced by Frahm and later formalized by Den Hartog, who analyzed the system as a two-degree-of-freedom model under harmonic excitation. Subsequent studies demonstrated that properly tuned absorbers significantly reduce structural response by generating an anti-resonance effect. Research by Abe and Fujino, Igusa and Xu, and others established design principles for optimal tuning and damping characteristics.

Despite its effectiveness, a single TMD is highly sensitive to parameter uncertainties such as variations in

structural frequency or damper properties. Even small mistuning can significantly reduce performance. To overcome this limitation, the concept of Multiple Tuned Mass Dampers (MTMDs) was proposed, where several dampers with slightly different dynamic characteristics are distributed over a frequency range. This configuration improves robustness and performance under broadband excitations such as earthquakes.

Previous investigations have shown that optimally designed MTMD systems provide improved reliability and efficiency compared with single TMDs. In the present work, the seismic response of a structure subjected to base excitation is analyzed with no damper, a single TMD, and MTMD systems consisting of 11 and 21 dampers. Performance comparison is based on maximum top-storey displacement obtained from linear time-history analysis using four earthquake records.

2. Structural model of MTMD

The structural system consists of a primary structure connected to *n* tuned mass dampers having different dynamic characteristics (Fig. 1). The main structure is defined by mass msm_sms, natural frequency ωs\omega_sωs, and damping ratio ξs\xi_sξs. Each TMD and the primary structure are modeled as single-degree-of-freedom (SDOF) systems, resulting in a total of n+1n+1n+1 degrees of freedom. The analysis is based on the following assumptions:

1. Structural natural frequencies are well separated.
2. Vibration suppression targets a specific dominant mode.
3. All dampers possess identical stiffness and damping constants.
4. MTMD natural frequencies are uniformly distributed around an average value.
5. The central TMD is perfectly tuned with the primary structure.

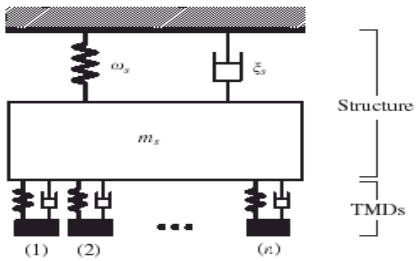


Figure 1: MTMD attached to the main system

The distribution of natural frequencies of the MTMD can be made either by varying the stiffness or mass of each TMD. However, manufacturing of TMD with uniform stiffness and damping constant is simpler than varying stiffness and damping properties (the mass remains unchanged). It is to be noted that MTMD's with identical dynamic characteristics are equivalent to a single TMD. The damping ratio and natural frequency of the equivalent single TMD are the same as those of individual MTMD. However, the mass is the sum of all the MTMD's mass. The system parameters used for the present optimization study are summarized as:

Let ω_T be the average frequency of all MTMD's (i.e. $\omega_T = \sum_{i=1}^{n} \frac{\omega_i}{n}$), where n be the total number of MTMD.

The natural frequency of i^{th} TMD is expressed as:

$$\omega_i = \omega_T \left[1 + \left(i - \frac{n+1}{2} \right) \left(\frac{\beta}{n-1} \right) \right] \quad (1)$$

Where parameter β is the non-dimensional frequency spacing of the MTMD defined as:

$$\beta = \frac{\omega_n - \omega_1}{\omega_T}$$

If k_T and c_T are the constant stiffness and damping of each TMD, then the mass and damping ratio of i^{th} TMD is expressed as:

$$m_i = \frac{k_T}{\omega_i^2}, \qquad \xi_i = \frac{c_T}{2 m_i \omega_i} = \left(\frac{c_T}{2 k_T} \right) \omega_i \quad (2)$$

The average damping ratio of MTMD is expressed as:

$$\xi_T = \sum_{i=1}^{n} \frac{\xi_i}{n} - \frac{\omega_T c_T}{2 k_T} \quad (3)$$

The ratio of total MTMD's mass to the main system's mass is defined as the mass ratio i.e.

$$\gamma = \frac{\sum_{i=1}^{n} m_i}{m_s} = \frac{m_T}{m_s} \quad (4)$$

Where m_T is the total mass of MTMD: and m_S is the mass of main system.
The constant stiffness and damping of each TMD may be evaluated using,

$$k_T = \frac{\gamma m_s}{\left(\sum_{i=1}^{n} \frac{1}{\omega_i^2} \right)}, \qquad c_T = \frac{2 \xi_T \gamma m_s}{\left(\omega_T \sum_{i=1}^{n} \frac{1}{\omega_i^2} \right)} \quad (5)$$

The ratio of the average frequency of the MTMD to the natural frequency of the main system is defined as the tuning frequency ratio; i.e. $f = \frac{\omega_T}{\omega_s}$ (6)

By using these parameters the MTMD are designed and applied to the building structure to reduce the structural vibration. The performance of MTMD can be characterized by using the terms effectiveness, robustness and redundancy as: Effectiveness is the reduction of the peak structural response under a given loading. For this study case it is the maximum top storey deflection. Robustness is the ability of the system to behave properly under slight mistuning or the parameter variations. Redundancy is the ability of the system to be effective when one or more of the dampers does not function.

3. Equation of motion of MTMD

The equation of motion of the system are derived from elementary dynamics and expressed in the matrix form as:

$$[M] \{\ddot{x}\} + [C] \{\dot{x}\} + [K] \{X\} = - [M] \{1\} X_g \quad (3.1)$$

Where, {X} is the displacement vector of the structural model,

i.e. $\{X\} = \{x_s, x_1, x_2 \ldots\ldots x_n\}^T$

X_g is the ground acceleration,

X_s is the displacement of main system relative to ground,

X_i (j=1,…,n) is the displacement of the i^{th} TMD relative to the ground.

{1} is the influence co-efficient matrix

{1} = {1, 1, 1, 1……..}^{T}

[M],[C]and[K] are mass, damping and stiffness matrices respectively

3.1 Numerical Study for Optimum Parameters

Joshi A. S. and Jangid R. S. (1997), have investigated the optimum parameters for stochastic response of the base excited main system with the MTMD. The criterion selected for optimality is minimization of the root mean square (r. m. s.) displacement of the main system. The base excitation is modeled as a stationary white noise random process; i.e.

$$S_{\ddot{x}_g}(\omega) = S_o, \quad -\infty \le \omega \le \infty \tag{3.1.1}$$

The effectiveness of the MTMD is defined by its equivalent damping added to the main system. The equivalent damping of the MTMD system is expressed by

$$\xi_{eq} = \frac{\pi S_o}{2\omega_s^{\,3}\sigma_{x_s}^{\,2}} - \xi_s \tag{3.1.2}$$

Where, $\sigma_{x_s}^{\,2}$ is the mean square displacement of the main system

The effect of the MTMD is considered in terms of the damping added to the main system. Note that ξ_{eq} is a measure of the effectiveness of the MTMD system, and that a positive value indicates that the MTMD system is effective in reducing the dynamic response of the main system. The ratio (R) of r.m.s. displacement of the main system with the MTMD to that without the MTMD is given by

$$R = \sqrt{\frac{\xi_s}{\xi_s + \xi_{eq}}} \tag{3.1.3}$$

The optimum parameters are obtained using a numerical searching procedure. For a given ξ_s , γ and n the parameters of the MTMD (i.e., ξ_T ,β and f) are varied such that the r.m.s. displacement of the main system attains the minimum value. The constraints applied on the values of parameters ξ_T ,β and *f* for the optimization study are as follows

$$0 \le \xi_T < 1, \quad 0 \le \beta < 2, \quad f > 0$$

The above conditions satisfy the conditions that,

1) The natural frequencies of the TMD's are positive real and

2) The TMD's are under damped.

These parameters are further used for analysis of the building. The figures below show the variation of the parameters with each other.

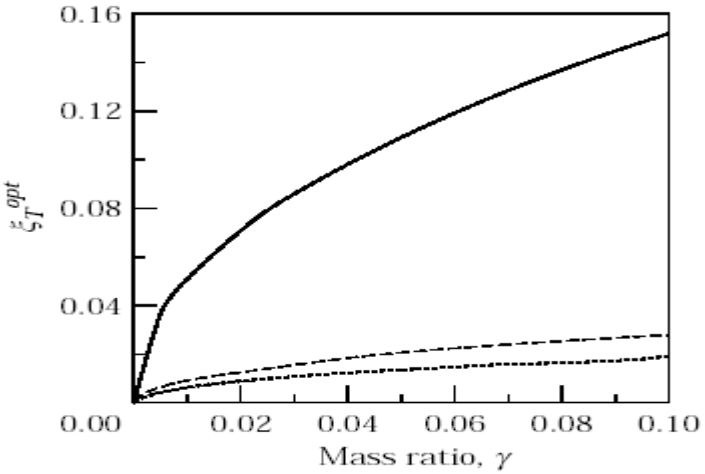


Figure 2: Variation of the optimum MTMD damping, ξ_T^{opt}, verses the mass ratio, γ, for f = f^{opt}, $\beta = \beta^{opt}$ and ξ_s = 0. ——, n = 1; - - - -, n = 11; ·····, n = 21

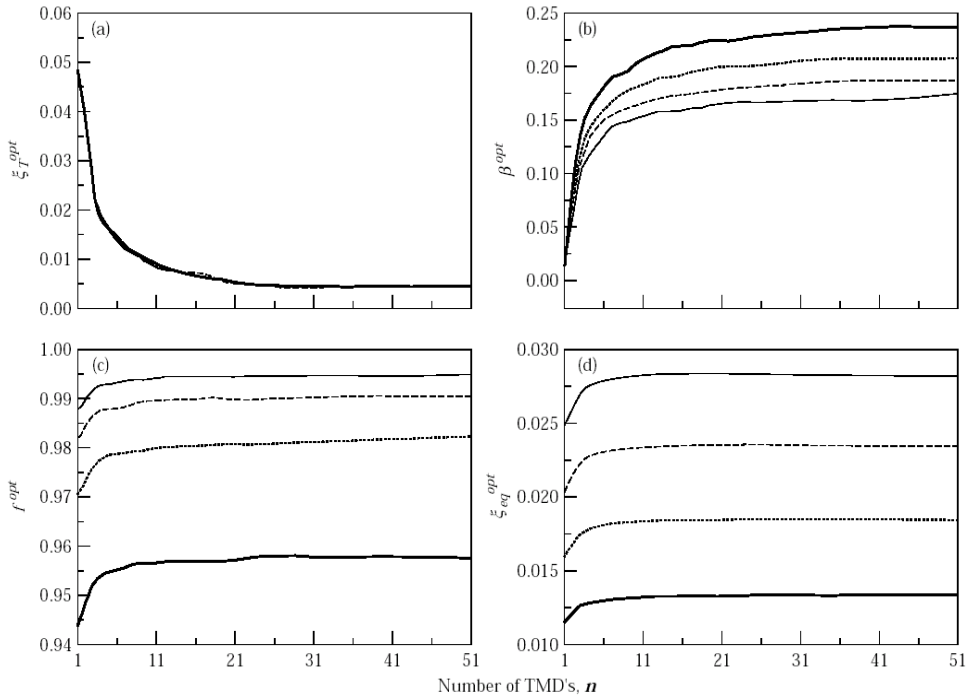


Figure 3: Variation of the optimum parameters (a) ξ_T^{opt},(b) β^{opt},(c) f^{opt} and (d) ξ_{eq}^{opt} verses umber of tuned mass dampers, for γ = 1% ——, ξ_s = 0%; - - - -, ξ_s = 2%; ·····, ξ_s = 5%; -------- ξ_s = 10%.

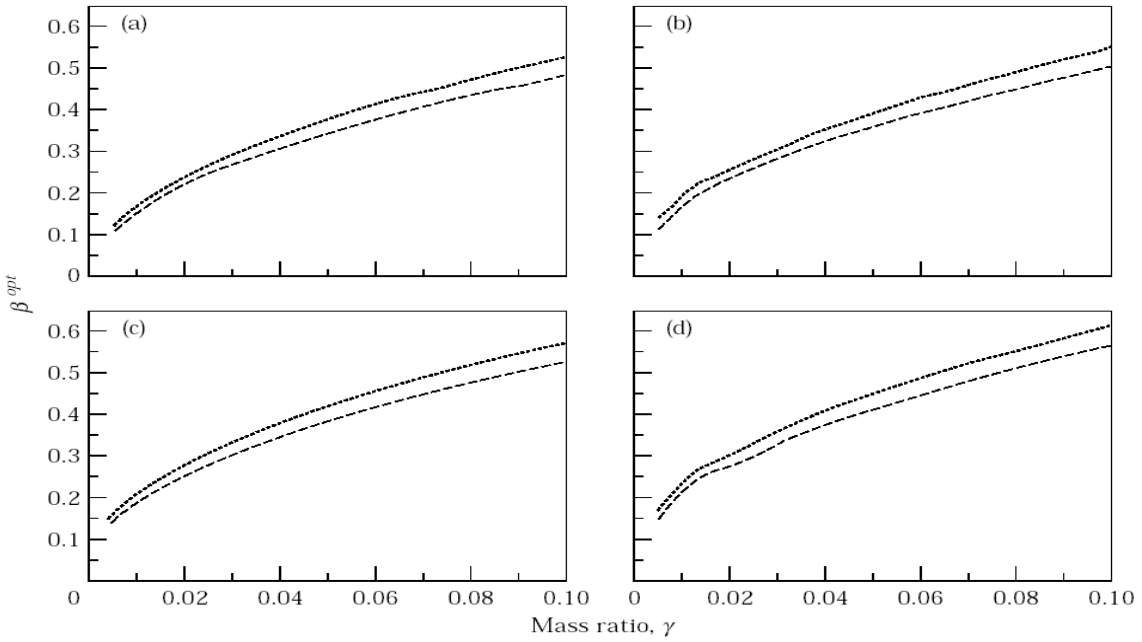


Figure 4: Variation of the optimum frequency range of the MTMD, β^{opt}, verses the mass ratio, γ, for f = f^{opt}, ξ_T = ξ_T^{opt}, (a) ξ_s = 0%, (b) ξ_s = 2%, (c) ξ_s = 5%, (d) ξ_s =10%, -------, n = 11; ·····, n = 21

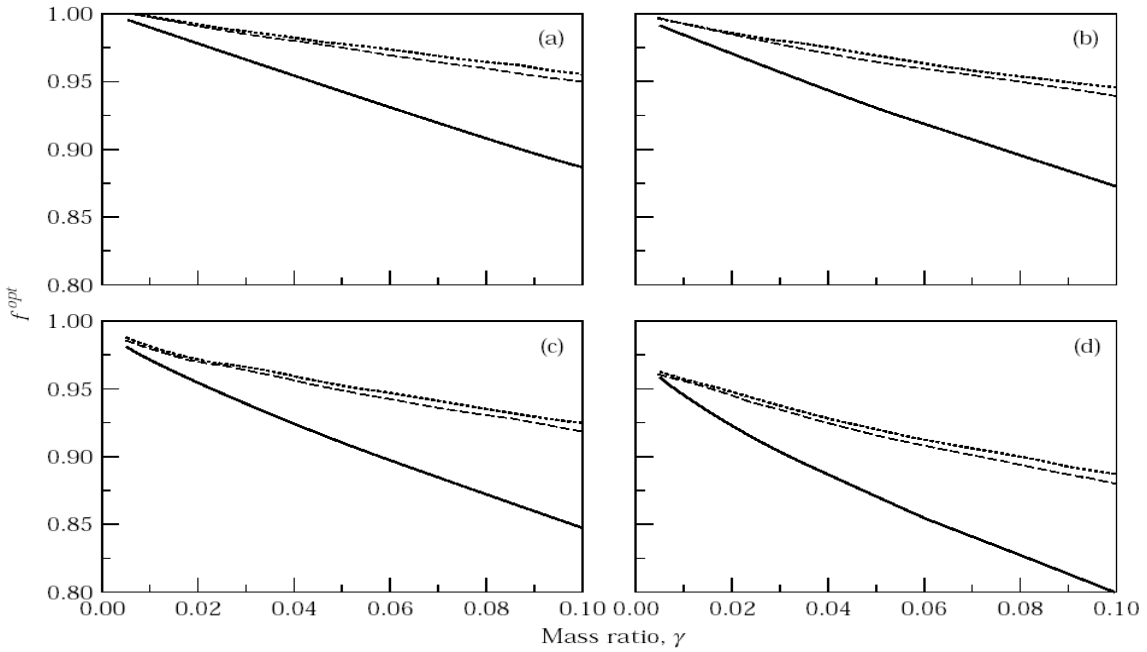


Figure 5: Variation of the optimum tuning frequency of the MTMD, f^{opt}, verses the mass ratio, γ, for β = β^{opt}, ξ_T = ξ_T^{opt},(a) ξ_s = 0%, (b) ξ_s = 2%, (c) ξ_s = 5%, (d) ξ_s =10%, ——, n = 1;----------, n = 11; ·····,n = 21.

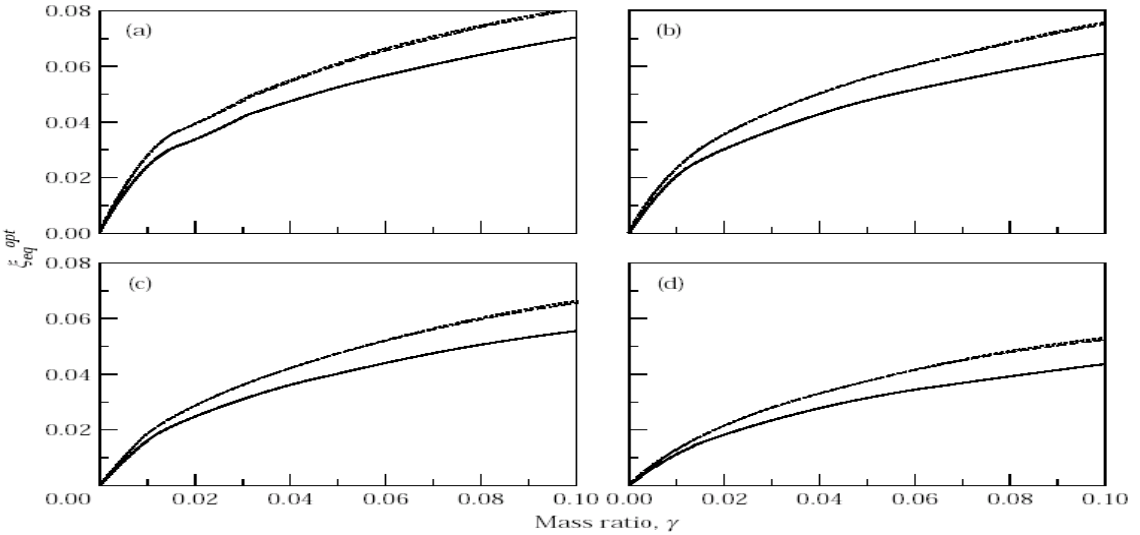


Figure 6: Variation of the optimum equivalent damping ratio ξ_T^{opt}, verses the mass ratio, γ, for $\beta = \beta^{opt}$, $\xi_T = \xi_T^{opt}$ and f = f^{opt} (a) ξ_s = 0%, (b) ξ_s = 2%, (c) ξ_s = 5%, (d) ξ_s =10%, ———, n = 1; ------, n = 11; ·····, n =21

4. Illustrative Example

4.1 Analysis of Multistory Building Frame

A single-bay four-storey RCC plane frame is modeled as an N-storey shear building. Tuned mass dampers are installed at the top storey, and both structure and dampers are idealized as SDOF systems. Linear time-history analysis is conducted for cases with and without dampers under gravity and earthquake loading. Three damper configurations are considered: n=1n=1n=1, n=11n=11n=11, and n=21n=21n=21. Damper frequencies are distributed around the fundamental frequency of the building to control the dominant vibration mode. Analyses are performed for building mass ratios of 1% and 2% and structural damping ratios of 0%, 2%, and 5%. The building properties, modal characteristics, and analytical results are presented in Tables 1–10.

Table 1: Total assembled joint masses in global coordinates

	Ux	Uy	Uz	Rx	Ry	Rz
TOTAL	36.6626	0	36.6626	0	0	0

Table 2: Modal period and frequencies

Mode	Period (sec)	FREQUENCY (rad/sec)
1	0.576094	10.90652
2	0.186511	33.688055
3	0.108946	57.672405
4	0.082034	76.592057
5	0.049757	126.278348

Based on above properties of the building the dampers are designed and the results of the analysis are presented below, which are discussed later.

Table 3: Displacement of various floors of the building

Floor	Displacement (mm)			
	No damper	n=1	n=11	n=21
4	109.51	103.38	103.38	103.15
3	95.22	89.49	89.49	89.28
2	66.18	61.79	61.79	61.64
1	28.99	27.85	27.85	27.81

Table 4: Acceleration of various floors of the building

Floor	Acceleration (mm/s2)			
	No damper	n=1	n=11	n=21
4	17200	16770	16770	16750
3	16200	15550	15550	15530
2	14700	14070	14070	14040
1	8751	8914	8914	8914

Table 5: Displacement of top floor for mass ratio = 1% and building damping = 0%

Name of Earthquake	Displacement of top floor (mm)			
	No damper	n=1	n=11	n=21
El-centro	203.20	199.542	199.543	199.540
Northridge	397.35	392.54	392.56	392.36
sakaria	178.71	162.40	162.41	161.79
holli	0.1861	0.1811	0.1811	0.1810

Table 6: Displacement of top floor for mass ratio = 1% and building damping = 2%

Name of Earthquake	Displacement of top floor (mm)			
	No damper	n=1	n=11	n=21
El-centro	124.56	122.712	122.713	122.711
Northridge	307.94	304.81	304.82	304.71
sakaria	109.51	103.38	103.37	103.15
holli	0.1075	0.1073	0.1073	0.1074

Table 7: Displacement of top floor for mass ratio = 2% and building damping = 5%

Name of Earthquake	Displacement of top floor (mm)			
	No damper	n=1	n=11	n=21
El-centro	92.18	92.30	92.31	92.31
Northridge	215.39	212.25	212.23	212.15
sakaria	82.32	79.01	79.01	78.94
holli	0.1320	0.1284	0.1284	0.1282

Table 8: Displacement of various floors of the building for redundancy check

Floor	Displacement (mm)			
	No damper	n=1	n=11	n=10
top	109.51	103.38	103.38	103.90

Table 9: Displacement of various floors of the building for optimal placement

Floor	Displacement (mm)		
	No damper	Dampers placed at top floor n=11	Dampers placed at each floor n=11
top	109.51	103.38	105.82

Table 10: Displacement of various floors of the building for robustness check

Floor	Displacement (mm)		
	No damper	n=11(1)	n=11(2)
top	109.51	103.38	103.39

4.2 Discussion of Results

Tables 3 and 4 indicate noticeable reductions in displacement and acceleration due to the inclusion of dampers. Comparisons under different earthquake records show that installing dampers at the top storey generally decreases peak structural response, although the degree of reduction depends on ground motion characteristics.

The performance of the 11-damper system is comparable to that of a single TMD, while the 21-damper system provides slightly improved vibration attenuation. MTMD systems demonstrate significant redundancy; failure of an individual damper produces negligible change in overall response (Table 8).

Placement studies reveal that locating dampers at the top floor is more effective than distributing them across storeys (Table 9). Robustness evaluation (Table 10) shows minimal sensitivity of MTMD performance to frequency estimation errors, confirming improved reliability compared with single TMD systems.

Overall, increased mass ratio enhances vibration reduction, whereas higher inherent structural damping reduces relative damper effectiveness. The results confirm that optimally designed MTMD systems effectively control seismic response.

5. Conclusions

This study investigated the effectiveness of Multiple Tuned Mass Dampers for seismic control of multi-storey buildings. Based on analytical and numerical results, the following conclusions are drawn:

1. Optimally designed MTMD systems provide improved vibration reduction compared with a single TMD.
2. The 11-damper configuration performs similarly to a single TMD, while the 21-damper system offers slightly better response control.
3. Damper effectiveness increases with mass ratio and decreases as inherent structural damping increases.
4. MTMD systems exhibit strong robustness against frequency mistuning.

5. Redundancy is a key advantage, as failure of individual dampers does not significantly degrade performance.
6. Smaller individual damper sizes improve constructability, installation flexibility, and maintenance.
7. Required optimal damping ratios for MTMDs are smaller than those of single TMD systems.
8. Response reduction depends on characteristics of the input ground motion.

List of notations:

c_i = damping of the i^{th} TMD
c_s = damping of main system
c_T = damping constant of each TMD
[C] = damping matrix
f = tuning frequency ratio
k_j = stiffness of the i^{th} TMD
k_s = stiffness of main system
k_T = constant stiffness of each TMD
[K] = stiffness matrix
m_i = mass of the i^{th} TMD
m_T = total mass of MTMD
m_s = mass of the main system
[M] = mass matrix
n = number of the tuned mass dampers
R = the response ratio
$S_{\ddot{x}_g}(\omega)$ = PSDF of the base acceleration
S_o = intensity of the white noise excitation
$S_{x_g}(\omega)$ = PSDF of the displacement of main system
x_s = displacement of the main system relative to ground
x_i = displacement of the i^{th} tuned mass damper relative to ground
$\ddot{x}_g$ = base acceleration
X(ω) = amplitude vector of the steady state response
β = non-dimensional frequency bandwidth of MTMD system
γ = mass ratio
ω = circular frequency
ω_T = average natural frequency of the MTMD
ω_i = natural frequency of the i^{th} TMD
ω_s = natural frequency of the main system
ξ_s = damping ratio of the main system
ξ_i = damping ratio of the i^{th} TMD
ξ_T = average damping ratio of MTMD
ξ_{eq} = equivalent damping added to main system by MTMD
x_s = mean square displacement of the main system
$\sigma^2_{x_s}$ = mean square response of the main system
TMD = Tuned mass damper
MTMD = Multiple tuned mass damper

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