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Synthesis Study of Air-to-Air Missile Guidance Methods for Long-Range Interception of Separating Targets

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Abstract

The long-range interception of group aerial targets that perform separation maneuvers presents a critical challenge for modern air-to-air missile guidance systems. Sudden target splitting leads to abrupt variations in line-of-sight dynamics, increased target uncertainty, and heightened maneuvering demands on the interceptor. Conventional guidance laws, particularly classical proportional navigation, may experience degraded performance under such conditions. This paper presents a synthesis study of air-to-air missile guidance methods applicable to long-range engagements involving separating targets. Classical proportional navigation-based approaches, augmented and adaptive guidance laws, optimal control-based methods, and intelligent guidance strategies are systematically reviewed and analyzed. Their effectiveness, limitations, and applicability to target separation scenarios are discussed and compared. The study aims to provide a comprehensive reference for the selection, design, and simulation-based evaluation of guidance laws suitable for countering target separation tactics in long-range air-to-air engagements.
Keywords: Air-to-air missile, separating targets, guidance law OPPN, long-range interception, proportional navigation.

1. Introduction

Modern air combat environments increasingly involve group aerial targets employing coordinated tactics to degrade interception effectiveness. One of the most challenging tactics is target separation, in which a group target splits into multiple independent objects at long engagement ranges. Such maneuvers introduce abrupt changes in engagement geometry, line-of-sight (LOS) dynamics, and target intent, significantly complicating the guidance task of air-to-air missiles [1-3].

In long-range air-to-air engagements, guidance systems must operate under strict constraints on available maneuver time, propulsion energy, and sensor accuracy. When target separation occurs, the missile seeker is required not only to maintain stable tracking but also to rapidly adapt its guidance commands to sudden variations in LOS rate and target acceleration. Conventional guidance laws, particularly classical proportional navigation (PN) [3-8], are known for their simplicity and robustness; however, their performance may deteriorate when facing highly maneuvering or separating targets, potentially resulting in increased miss distance or loss of target lock. To address these challenges, numerous guidance methods have been developed and reported in the literature. Augmented and biased proportional navigation laws have been proposed to improve performance against maneuvering targets by incorporating target acceleration estimates or predefined correction terms. Adaptive guidance approaches further enhance robustness by adjusting guidance parameters in real time according to engagement conditions. In parallel, optimal control-based guidance laws derived from calculus of variations and dynamic programming offer theoretically optimal solutions for long-

range interception problems, although their practical implementation is often limited by computational complexity and sensitivity to modeling uncertainties [9-12].

More recently, intelligent guidance methods based on artificial intelligence techniques, such as neural networks and reinforcement learning, have attracted increasing attention [12-15]. These approaches demonstrate strong potential in handling nonlinear dynamics, target uncertainty, and complex behaviors such as target separation. Hybrid guidance architectures that combine classical guidance laws with intelligent correction terms have emerged as a promising compromise between performance and real-time feasibility.

Despite extensive research efforts, a unified and systematic synthesis of air-to-air missile guidance methods specifically addressing long-range interception of separating targets remains limited. Most existing studies focus on individual guidance laws or specific engagement scenarios, making it difficult to objectively compare their effectiveness and applicability.

2. Optimal Guidance Law with Maneuver Compensation

For the general situation, the optimal control law synthesis problem uses linear models defined in state space, which can be expressed by the following differential equation:

$$\dot{x} = f\left(x(t), u(t), t\right) \tag{1}$$

Where: $x(t)$ is the state vector of dimension n_x, $u(t)$ is the input (control) vector of dimension n_u.

To synthesize the optimal control law based on the normal acceleration of the missile, it is necessary to minimize the following quality index function [13]:

$$J = \int_0^{t_k} \Phi_T\left[x(t), u(t), t\right]dt + \Phi_K\left[x(t_k), u(t_k), t_k\right] \tag{2}$$

Where: $\Phi_T[\ldots], \Phi_K[\ldots]$ -The general expression under the integral sign corresponds to the instantaneous and terminal components. The control will be optimal if J is always minimal in the time interval $[0; t_k]$.

Based on the above problem of synthesizing the guidance law for air-to-air missiles to the group target, the kinematic model of the AAM missile clearly must take into account the relative displacement between the interceptor-AAM and the targets in the group when the antenna wave section of the homing head on the missile is oriented towards the group target. In other words, it is necessary to orient the interceptor's velocity vector to be inclined at an angle other than the orbital tilt angle of the conventional PN guidance method so that it can be obtained separately from radial velocities (closing velocity V_c) corresponding to each target in the group. This can ensure the ability of the homing head to distinguish each target according to the Doppler frequency (F_D) received in the reflected signal and identify them through secondary modulation of the reflected signal.

The kinematic correlation describing the relative motion between an interceptor missile and a target in the group is illustrated in Figure 1. Fig.1 shows the interceptor missile is guided to the target according to the PN [7] and according to the proposed guidance law.

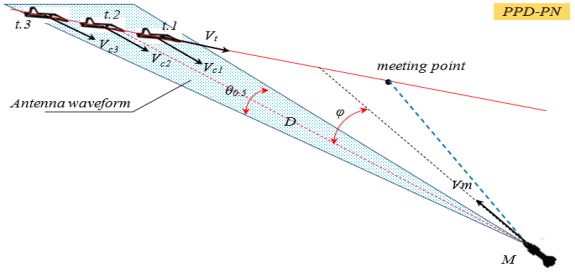


Figure 1. The kinematic of relative motion between missile and target when using the PN guidance law and proposed guidance law.

Where: $\bar{V}_T$, $\bar{V}_M$ - Velocity vectors of the target and missile; D - The distance between target - missile; ω - angular velocity of the line of sight missile-target in the horizontal plane; φ - missile orientation angle in the horizontal plane; q - target identification cone angle.

According to [6], we have an equation describing the dependence of the angular velocity of the line of sight with the normal acceleration of the missile expressed in the form of the following equation:

$$\dot{\omega} = -\frac{2\dot{D}}{D}\omega + \frac{1}{D}(W_T - W_M); \quad \omega(0) = \omega_0 \tag{3}$$

Where: W_T, W_M - The normal acceleration of the target and the missile in the horizontal plane, respectively. To synthesize the optimal guiding law for interceptor missiles to ensure the requirements for discriminating targets in the group by Dopler frequency. We need to add equation (3) to the differential equations that describe direct control of the interceptor missile's trajectory as follows:

$$\dot{\omega}_r = 0; \quad \omega_r(0) = \frac{\lambda}{2\Delta L_{\min}}\omega_q \tag{4}$$

Where: ω_r - The angular velocity of the line-of-sight required to produce the deviation of the orbital tilt angle missile trajectory.

ω_q - Angular velocity of the antenna lobe of the missile's homing head, rad/s.

$\Delta L_{\min}$ - Minimum resolution according to the distance of the homing head's radar, m.

λ - The wavelength of the detection pulse generator of the homing head's radar, m.

From the analysis of Fig.1, the expression for the angular velocity of the line-of-sight Missile-Target can be found as follows:

$$\omega = \frac{V_M \sin\varphi - V_T \sin q}{D} \tag{5}$$

From (5), the opening angle of the target recognition cone can be determined by the formula:

$$q = \arcsin\left(\frac{V_M \sin\varphi - \omega D}{V_T}\right) \approx \arcsin\left(\frac{V_M\varphi - \omega D}{V_T}\right) \tag{6}$$

Differentiate (6) concerning time, we have:

$$\dot{q} = \frac{V_M\varphi - \dot{\omega}D - \omega\dot{D}}{\sqrt{V_T^2 - (V_M\varphi - \omega D)^2}} \tag{7}$$

Substitute equation (3) into (7), we get:

$$\dot{q} = \frac{\dot{D}\omega + V_M\dot{\varphi} + W_M - W_T}{\sqrt{V_T^2 - (V_M\varphi - \omega D)^2}} \tag{8}$$

Or:

$$\dot{q} = C\left(\omega\dot{D} + V_T\dot{\varphi} + W_M - W_T\right) \tag{9}$$

In which:

$$C = \frac{1}{\sqrt{V_T^2 - (V_M\varphi - \omega D)^2}} \tag{10}$$

From equations (4), (9), and (10) together with geometric dynamics equations (3) it is possible to determine a general kinetic model, which is the basis for synthesizing the optimal guidance law for missiles to intercept the group target (or separating target). This guidance law allows the radar of the homing head to have the best conditions for radio observation, discrimination, identification of targets in the group, and detection of maneuvering targets (separating from the group) to decide to follow. In general, we have the following system of differential equations:

$$\begin{aligned}
&\dot{\omega}_i = -\frac{2\dot{D}_i}{D_i}\omega_i + \frac{1}{D_i}\left(W_M - W_{iT}\right) + \xi_\omega \\
&\omega_i(0) = \omega_{0i}; \quad \dot{\omega}_{ir} = 0; \quad \omega_{ir}(0) = \frac{\lambda}{2\Delta L_{\min}}\omega_q; \\
&\dot{q}_i = C\left(\omega_i D_i + V_M \varphi_i + W_M - W_{iT}\right); \\
&q_i(0) = q_{0i}; \quad \dot{q}_{ir} = 0; \quad q_{ir}(0) = const.
\end{aligned} \tag{11}$$

In which:

D_i - presents the relative distance between the missile and the "ith" target in the group.

W_M ;W_{iT} - relatively the normal acceleration of the missile and the ith target.

ω_i - the angular velocity of the ith line-of-sight Missile-Target.

ξ_ω - gause white noise with a given one-sided spectral density.

ω_{ir} - the required normal component for the angular velocity of the ith line-of-sight Missile-Target.

q_i - the radio observation angle for the second modulated signal, reflected from the ith target to the radar of the homing head.

q_{ir} - the required radio observation angle for the second modulated signal, reflected from the target to the radar of the homing head.

In the process of guiding a missile, the main control parameter is the normal acceleration of the interceptor. When synthesizing the guiding law, the normal acceleration must be optimal. It allows to guarantee of the conditions, related to the radio observation of the mobile radar for the reflected signals from the targets in the group, and to satisfy the requirement of minimum miss distance during the guided process. The control vector U is represented as a scalar as follows:

$$U = W_M \tag{12}$$

The required angular velocity of the line-of-sight Missile-Target (ω_r) and the required radio observation angle (q_r) (for the homing head's radar to have the ability to distinguish the reflected signal by Doppler frequency) are already required parameters for the missile guidance system. They form a vector of required phase coordinates of the form:

$$X_T = \begin{bmatrix} \omega_r & q_r \end{bmatrix}^T \tag{13}$$

It follows that the controlled parameters in the guidance system will be (ω and q), and then the vector of controlled phase coordinates has the form:

$$X_Y = \begin{bmatrix} \omega & q \end{bmatrix}^T \tag{14}$$

Since the control parameter is the missile's normal acceleration (W_M), and the controlled parameters are (ω and q), the efficiency matrix of the control signals ($\mathbf{B}_Y^T$) will gather the coefficients related to (W_M). The differential equations (3); (4); (7); (9) and (10) describe the change of the controlled parameters as (ω and q). From the optimal kinematic model (11), the effective matrix of the control signals $\mathbf{B}_Y^T$ will have the form:

$$\mathbf{B}_Y^T = \begin{bmatrix} -\frac{1}{D} & C \end{bmatrix} \tag{15}$$

According to (12) the control vector U has size $[1\times1]$ and the sizes of the matrices (13) – (15) are both $[1\times2]$, it is possible to define the penalty coefficient matrix for the tracking accuracy Q and the magnitudes of the signals K in the following form:

$$\mathbf{Q}_Y^T = \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} \tag{16}$$

$$\mathbf{K}=k \tag{17}$$

The result of minimizing the optimal criterion function (1) will have the form:

$$\mathbf{J} = M_Y \left\{ \begin{bmatrix} \omega_r - \omega \\ q_r - q \end{bmatrix}^T \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} \begin{bmatrix} \omega_r - \omega \\ q_r - q \end{bmatrix} + \int_0^{t_i} W_M^2 \, k dt \right\} \tag{18}$$

The optimal control law which minimizes the criterion function (1), is determined by the following formula:

$$U(t) = K^{-1} B_Y^T(t) Q \left[\hat{X}_T(t) - \hat{X}_Y(t) \right] \tag{19}$$

This is the control signal and the solution of the Bellman equation [13]:

Thus, after solving equation (19) and taking into account equations (12) – (16), we can get the optimal control law for the normal acceleration of the AAM:

$$W_{OPPN} = \frac{1}{k} \cdot \begin{bmatrix} -\frac{1}{D} \\ C \end{bmatrix}^T \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} \begin{bmatrix} \omega_r - \omega \\ q_r - q \end{bmatrix} = \frac{1}{k} \cdot \begin{bmatrix} -\frac{1}{D} & C \end{bmatrix} \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} \begin{bmatrix} \omega_r - \omega \\ q_r - q \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{kD} & \frac{C}{k} \end{bmatrix} \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} \begin{bmatrix} \omega_r - \omega \\ q_r - q \end{bmatrix} = \begin{bmatrix} -\frac{q_{11}}{kD} & \frac{Cq_{22}}{k} \end{bmatrix} \begin{bmatrix} \omega_r - \omega \\ q_r - q \end{bmatrix}$$

$$= -\frac{q_{11}}{kD}(\omega_r - \omega) + \frac{Cq_{22}}{k}(q_r - q)$$

$$\Rightarrow W_{OPPN} = -\frac{q_{11}}{kD}(q_r - \omega) + \frac{Cq_{22}}{k}(q_r - q) \tag{20}$$

Equation (20) is called the Optimal Proportional Preemptive Navigation law (OPPN) for a missile intercepting a group target or separating target. Based on the guidance law expression (20) we can see that, in the first part of the guidance law, it is possible to separately distinguish the targets in the group according to the Doppler frequency in the reflected signal received by the homing head's radar; in the second part, the missile is guided along a trajectory, in which its velocity vector always make with the line of sight Missile-Target a required deviation angle (q_r), which is necessary for sustained observation of the reflected signals containing their own Doppler frequency and secondary modulation, serving the problem of identifying the type of the target in the group.

3. Simulation and Evaluation

To evaluate the efficiency of the newly synthesized guidance law OPPN (20) compared with the guidance law PN [7], we perform a simulation on a computer with parameters according to Table 4. Simulation conditions (air situation) on the vertical plane of the two guidance laws are the same. Survey the interceptor missile's trajectory characteristics, the miss distance, and angle of reflection according to both guidance laws when intercepting a single target and investigate the effectiveness of the OPPN guidance law for separating targets.

Table 1. Parameters for Missile and target simulation

Missile				Target			
Initial velocity (m/s)	Horizontal distance (km)	Height (km)	Required observation angle (o)	Initial velocity (m/s)	Horizontal distance (km)	Height (km)	Orbital tilt angle (o)
1100	0	0	15	350	25	15	120

$\omega_q = 10\ rad/s$ – Angular velocity of the antenna lobe of the missile's homing head.

$\Delta L_{\min} = 150m$ – Minimum resolution according to the distance of the homing head's radar.

$\lambda = 0.03\ m$ – The wavelength of the detection pulse of the homing head's radar.

3.1. Comparative survey of the PN and OPPN guidance quality for the same single target.

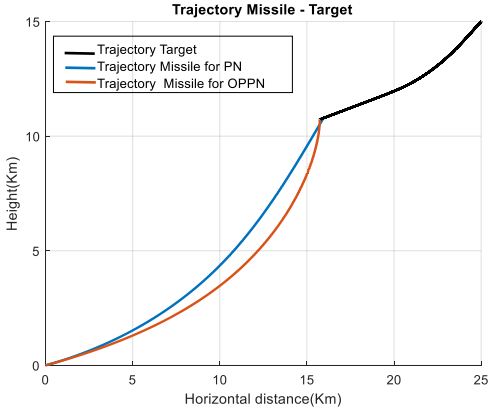


Figure 2: Missile trajectory according to PN and OPPN

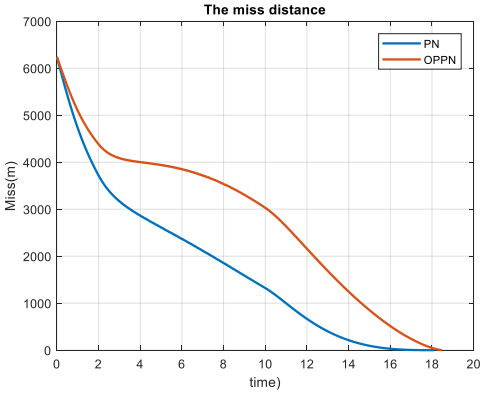


Figure 3. The miss distance according to PN and OPPN

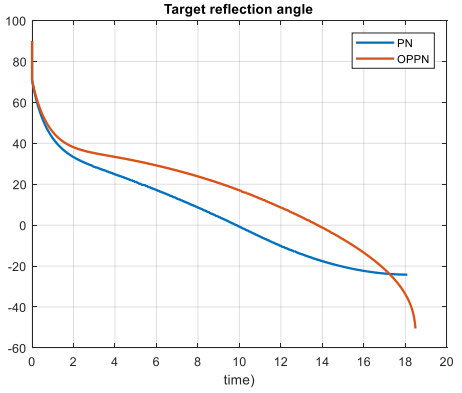


Figure 4. Change of target reflection angle according to PN and OPPN

Based on the survey results, the following comments and assessments can be drawn:

- For the same target variant, the missile trajectory guided by PN [2] method is straighter than that of OPPN, which is consistent with the result that instantaneous the miss distance PN decreases faster than OPPN, although both methods have the miss distance decreasing to zero in the vicinity of the intersection point.

- The radio observation angles (q_β) for the signal reflected from the target carrying the Doppler

frequency and the secondary modulation signal in the two methods have the law of decreasing over time. However, the OPPN method decreases more slowly than the PN method, which allows the missiles guided by OPPN to have a longer target observation time for discrimination.

Thus, when intercepting a single target, the trajectory of the missile and the miss distance according to the PN method is always better than that of OPPN. However, in the case of a group target, the OPPN method is more optimal in terms of setting the radio signal observation angle and observation time as stated in the research problem.

3.2. Survey of the OPPN guidance law for separating targets.

In case the separating target flies at the same speed and after 2s changes in normal acceleration.

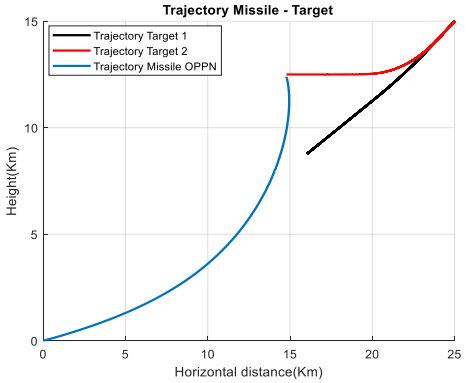


Figure 5: The missile trajectory intercepting the separating target

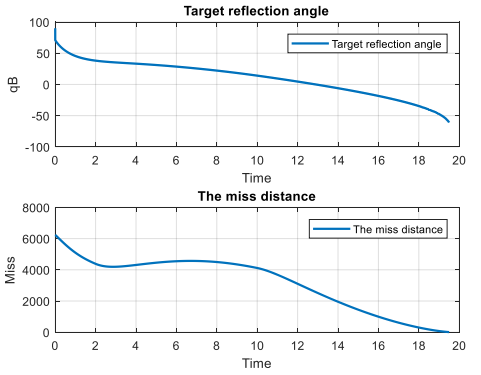


Figure 6: The change of target reflection angle and the miss distance

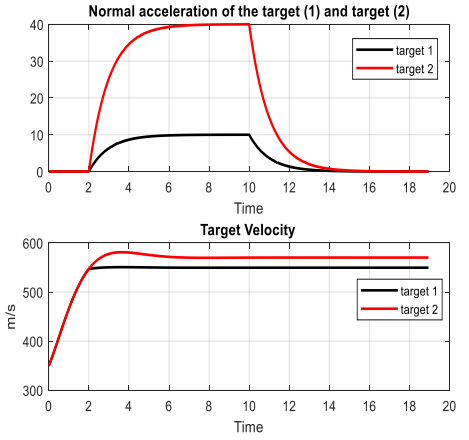


Figure 7: The change of velocity and normal acceleration of the carrying target (1) and the separating target (2)

3.3. Survey results of signal spectrum carrying Doppler frequency from the targets in the group

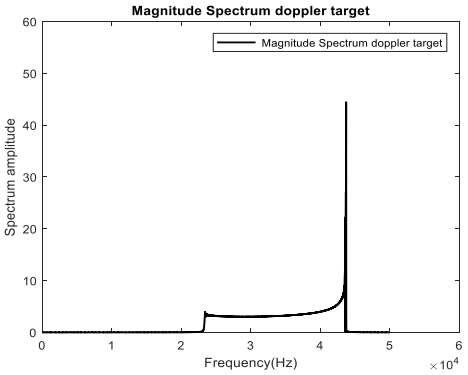


Figure 8. Doppler Spectrum signal reflected from an original target

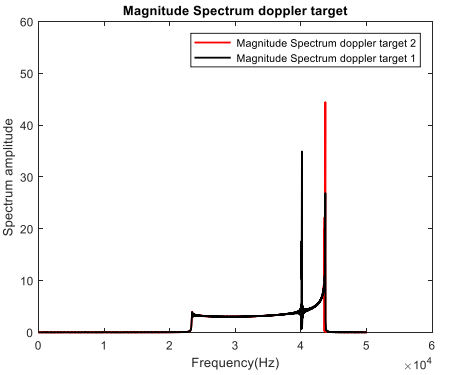


Figure 9. Doppler Spectrum signal reflected from separating target

Figure 8 and 9 shows the spectral portrait of the Doppler signal reflected from the carrying target (1), and Figure 9 shows the spectrum portrait of both the carrying target (1) and the separating target (2). Spectral portrait of the signal reflected from targets forms at the signal processor output. Accordingly, we can evaluate the spectral width of each signal, as the basis for building a narrowband Doppler filter.

4. Conclusion

This paper presents a synthesis of an optimal proportional navigation–based guidance law (OPPN) for air-to-air missiles, enabling effective discrimination among individual targets within a group. The proposed guidance strategy is applicable to medium- and long-range air-to-air missiles tasked with intercepting aircraft formations, single targets, or incoming counter-missiles. In this study, such targets are referred to as separating targets, representing a particularly dangerous class of super-maneuverable threats. To evaluate the effectiveness of the proposed OPPN guidance law, a series of simulations was conducted under scenarios involving dynamic state changes of separating targets. The results demonstrate that, during engagements with group or separating targets, missiles guided by the OPPN law can consistently distinguish individual targets based on Doppler frequency characteristics in the homing seeker's reflected signal, or alternatively classify target types through secondary modulation features. Furthermore, by integrating additional algorithms for estimating the line-of-sight angular rate between the missile and each target within the group, the OPPN-based guidance framework enables the missile to autonomously reassign its lock-on and engage the most threatening target. This capability is particularly critical when the group includes highly dangerous, super-maneuverable air-to-air missiles.

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Declaration of Conflicting Interests

The authors declare that there are no conflicts of interest regarding the research, authorship, or publication of this article.

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References

[1] А. Аганин, О. Васильев, and В. J. Р. Макаев, "*Распознавание воздушной цели класса «самолет с винтовым двигателем»,*" no. 8, pp. 69-73, 2001.

[2] А. Богданов, В. Голубенко, А. Княжев, and А. J. Р. Филонов, *"Алгоритм совместного траекторного сопровождения-распознавания функционального назначения самолетов, летящих в сомкнутом боевом порядке,"* no. 10, pp. 98-103, 2008.

[3] В. Макаев and О. J. Р. Васильев, *"Метод радиолокационного распознавания воздушной цели по турбинному эффекту,"* vol. 11, pp. 30-33, 2000.

[4] А. Богданов, О. Васильев, В. Голубенко, С. Маняшин, and А. J. И. Р. а. н. Т. и. с. у. Филонов, *"Методика построения динамических моделей радиальных скоростей и ускорений пары воздушных целей, летящих в сомкнутом боевом порядке,"* no. 4, pp. 137-147, 2007.

[5] О. Васильев, В. Абатуров, Р. Потапов, А. Ситников, and С. J. Р. Коротков, *"Особенности распознавания воздушных целей в бортовых РЛС при длительной когерентной обработке сигналов,"* no. 2, pp. 43-54, 2011.

[6] М. Ярлыков, А. Богачев, В. Меркулов, and В. J. М. Р. Дрогалин, *"Радиоэлектронные комплексы навигации, прицеливания и управления вооружением летательных аппаратов. Т. 1. Теоретические основы./Под ред. МС Ярлыкова,"* vol. 1, p. 504, 2012.

[7] P. Zarchan, *Tactical and strategic missile guidance*. American Institute of Aeronautics and Astronautics, Inc., 2012.

[8] S. A. Murtaugh and H. E. J. I. s. Criel, *"Fundamentals of proportional navigation,"* vol. 3, no. 12, pp. 75-85, 1966.

[9] N. F. Palumbo, R. A. Blauwkamp, and J. M. J. J. H. A. T. D. Lloyd, *"Basic principles of homing guidance,"* vol. 29, no. 1, pp. 25-41, 2010.

[10] G. M. Siouris, *Missile guidance and control systems*. Springer Science & Business Media, 2004.

[11] S. Vathsal and A. J. D. S. J. Sarkar, *"Current trends in tactical missile guidance,"* vol. 55, no. 3, p. 265, 2005.

[12] L.P. Tsao and C.S. Lin, *"New optimal guidance law for short-range homing missiles,"* in proc natl sci count repub china part a physscieng, 2000, vol. 24, no. 6, pp. 422-426.

[13] D. E. Kirk, *Optimal control theory: an introduction*. Courier Corporation, 2004.

[14] О. J. Р. Викулов, *"Траекторное управление наведением в активной радиолокационной системе самонаведения,"* vol. 11, pp. 81-85, 1995.

[15] М. Черных, О. Васильев, А. Богданов, А. Савельев, and В. J. Р. Макаев, *"Экспериментальные исследования информационных свойств когерентных радиолокационных сигналов,"* no. 3, p. 47, 2000.