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Synthesis of a Range-Measuring and Angle-Measuring Channel Filtering Algorithm to Estimate the Acceleration of a Maneuvering Air Target

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Abstract

Accurate estimation of acceleration is a critical requirement in modern air target tracking systems, particularly for maneuvering targets subject to high dynamics and abrupt changes in motion. Traditional tracking algorithms based solely on kinematic state estimation often exhibit degraded performance when applied to highly maneuverable air targets, especially in the presence of measurement noise and nonlinear observation models.

This paper presents the synthesis of a combined filtering algorithm utilizing both range-measuring and angle-measuring channels to estimate the acceleration of a maneuvering air target. The proposed approach exploits the complementary characteristics of range and angular measurements obtained from radar sensors to enhance observability and estimation accuracy. A unified dynamic model of the air target is developed, incorporating acceleration as an augmented state variable. Based on this model, a nonlinear filtering structure is synthesized to process range and angular information jointly.

Simulation results demonstrate that the proposed algorithm significantly improves acceleration estimation accuracy compared to single-channel filtering methods, particularly under intensive target maneuvers and noisy measurement conditions. The developed algorithm is suitable for application in missile guidance systems, radar tracking, and air defense fire control systems.

Keywords: *Maneuvering air target, acceleration estimation, range measurement, angle measurement, nonlinear filtering, radar tracking.*

1. Introduction

The continuous evolution of modern air threats, including highly maneuverable aircraft, unmanned aerial vehicles, and tactical missiles, has significantly increased the requirements for accuracy and robustness of air target tracking systems [1], [2]. In air defense and missile guidance applications, precise estimation of target kinematic parameters—especially acceleration—is essential for trajectory prediction, guidance law synthesis, and interception performance [3], [4].

Conventional radar tracking systems primarily rely on range and angular measurements to estimate the position and velocity of air targets. Classical filtering techniques, such as the Kalman filter and its extensions, provide optimal state estimation under linear dynamics and Gaussian noise assumptions [5]. However, when applied to maneuvering air targets, these methods often suffer from performance degradation due to model mismatch between assumed constant-velocity or constant-acceleration models and actual target motion [6].

Acceleration estimation plays a critical role in addressing this challenge, as acceleration represents the primary indicator of target maneuver intent [7]. Since radar sensors cannot directly measure acceleration, it must be inferred indirectly from noisy range and angle measurements. This inference problem is further complicated by the nonlinear relationship between angular measurements and Cartesian target states, leading to reduced observability and increased estimation errors during high-g maneuvers [8], [9].

Numerous approaches have been proposed to improve maneuvering target tracking, including adaptive filters, interacting multiple model (IMM) algorithms, and nonlinear estimators such as the extended Kalman filter (EKF) and unscented Kalman filter (UKF) [10]–[12]. Although these methods enhance tracking performance to some extent, their effectiveness strongly depends on accurate maneuver modeling and sufficient measurement information. In particular, filters relying on a single measurement channel—either range-only or angle-only—often fail to provide reliable acceleration estimates under intense maneuvering conditions [13].

To overcome these limitations, joint processing of range-measuring and angle-measuring channels has emerged as an effective solution for improving estimation accuracy and robustness [14]. By exploiting the complementary sensitivities of range and angular measurements, it becomes possible to enhance state observability and extract acceleration information more effectively, even in the presence of strong nonlinearities and measurement noise.

Motivated by these considerations, this paper presents the synthesis of a combined filtering algorithm that simultaneously utilizes range and angle measurement channels to estimate the acceleration of a maneuvering air target. The proposed approach incorporates acceleration as an augmented state variable and applies nonlinear filtering techniques to jointly process measurement information. The resulting algorithm is particularly suitable for applications in radar tracking systems, missile guidance, and air defense fire control, where high-accuracy acceleration estimation is required under complex target maneuvers.

2. Filtering Algorithm for Acceleration Estimation

Consider a maneuvering air target observed by a radar sensor providing range and azimuth angle measurements. The objective is to estimate the target acceleration based on noisy and nonlinear measurements obtained from both range-measuring and angle-measuring channels.

Let the discrete-time state vector at time step k be defined as:

$$\mathbf{x}_k = \left[x_k \;\; y_k \;\; v_{x,k} \;\; v_{y,k} \;\; a_{x,k} \;\; a_{y,k} \right]^T \tag{1}$$

where x_k, y_k denote target position, $v_{x,k}$, $v_{y,k}$ denote velocity, $a_{x,k}$, $a_{y,k}$ and k represent acceleration components.

The estimation problem consists of determining x_k, with particular emphasis on the acceleration components, using sequential radar measurements.

Assuming a constant-acceleration motion model with random maneuver excitation, the discrete-time state equation is given by

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{G}\mathbf{w}_k \tag{2}$$

where $\mathbf{w}_k$ is zero-mean Gaussian process noise accounting for maneuver uncertainty.

The state transition matrix F is defined as

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & T & 0 & \frac{T^2}{2} & 0 \\ 0 & 1 & 0 & T & 0 & \frac{T^2}{2} \\ 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where T is the sampling interval.

The radar measurement vector is defined as

$$\mathbf{z}_k = \begin{bmatrix} r_k \\ \theta_k \end{bmatrix} \tag{4}$$

with the nonlinear measurement equations

$$r_k = \sqrt{x_k^2 + y_k^2} + v_{r,k} \tag{5}$$

$$\theta_k = \arctan\left(\frac{y_k}{x_k}\right) + v_{\theta,k} \tag{6}$$

where $v_{r,k}$ and $v_{\theta,k}$ are zero-mean Gaussian measurement noises.

Due to the nonlinear nature of the measurement equations, linearization around the predicted state is performed. The measurement Jacobian matrix H_k is given by

$$\mathbf{H}_k = \begin{bmatrix} \frac{x_k}{r_k} & \frac{y_k}{r_k} & 0 & 0 & 0 & 0 \\ -\frac{y_k}{r_k^2} & \frac{x_k}{r_k^2} & 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

This Jacobian illustrates that acceleration information is indirectly embedded through the temporal evolution of position and velocity states.

The proposed filtering algorithm jointly processes range and angle measurements within a unified nonlinear filtering framework. The algorithm consists of the following steps:

Prediction:

$$\mathbf{x}_{k|k-1} = \mathbf{F}\mathbf{x}_{k-1|k-1} \tag{8}$$

Update:

$$\mathbf{P}_{k|k-1} = \mathbf{F}\mathbf{P}_{k-1|k-1}\mathbf{F}^T + \mathbf{Q} \tag{9}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\left(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}\right)^{-1} \tag{10}$$

$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{K}_k\left(\mathbf{z}_k - \hat{\mathbf{z}}_k\right) \tag{11}$$

where Q and R are the process and measurement noise covariance matrices, respectively.

$$\hat{\mathbf{z}}_k = \begin{bmatrix} \sqrt{\hat{x}_k^2 + \hat{y}_k^2} \\ \arctan\left(\frac{\hat{y}_k}{\hat{x}_k}\right) \end{bmatrix} \tag{12}$$

$$\hat{\mathbf{a}}_k = \begin{bmatrix} \hat{a}_{x,k} \\ \hat{a}_{y,k} \end{bmatrix} \tag{13}$$

Unlike conventional tracking filters, the proposed approach explicitly includes acceleration as a state variable. The joint utilization of range and angular measurements enhances observability of acceleration through correlated corrections of position and velocity states.

As a result, the filter can rapidly capture changes in target acceleration, even during abrupt maneuvers, without relying on heuristic maneuver detection or model switching.

3. Simulation and Evaluation

To evaluate the performance of the proposed range–angle combined filtering algorithm, numerical simulations were conducted for a maneuvering air target tracked by a ground-based radar. The target motion was modeled in a two-dimensional Cartesian coordinate system, with acceleration treated as an augmented state variable.

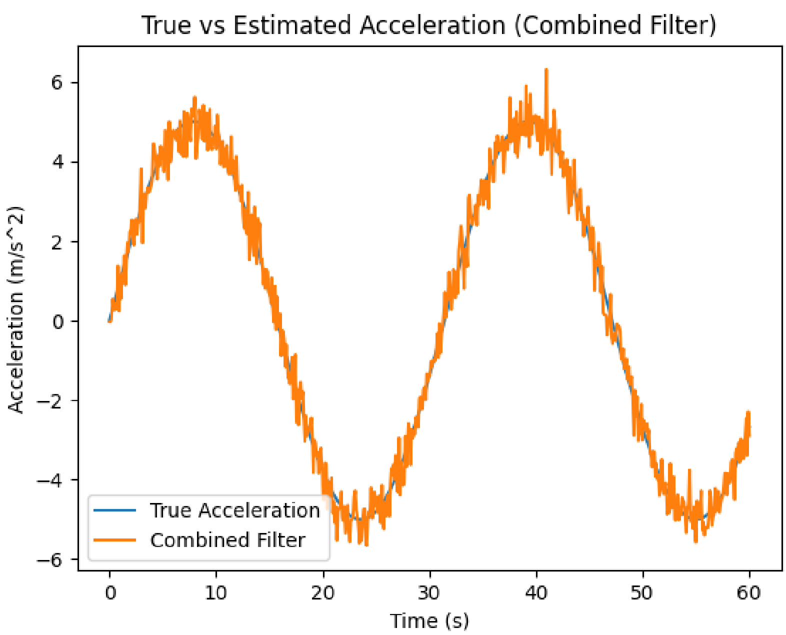

Figure 1. True vs Estimated Acceleration (Combined Filter)

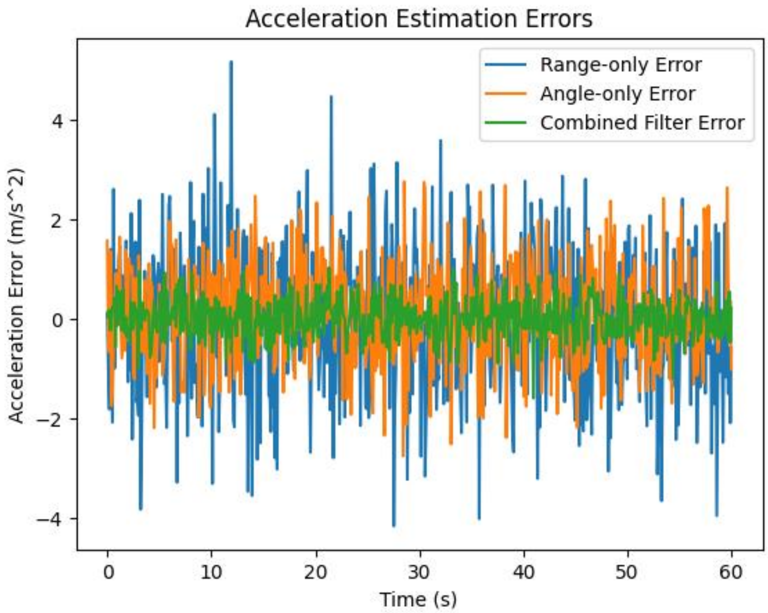

Figure 2. Acceleration Estimation Errors

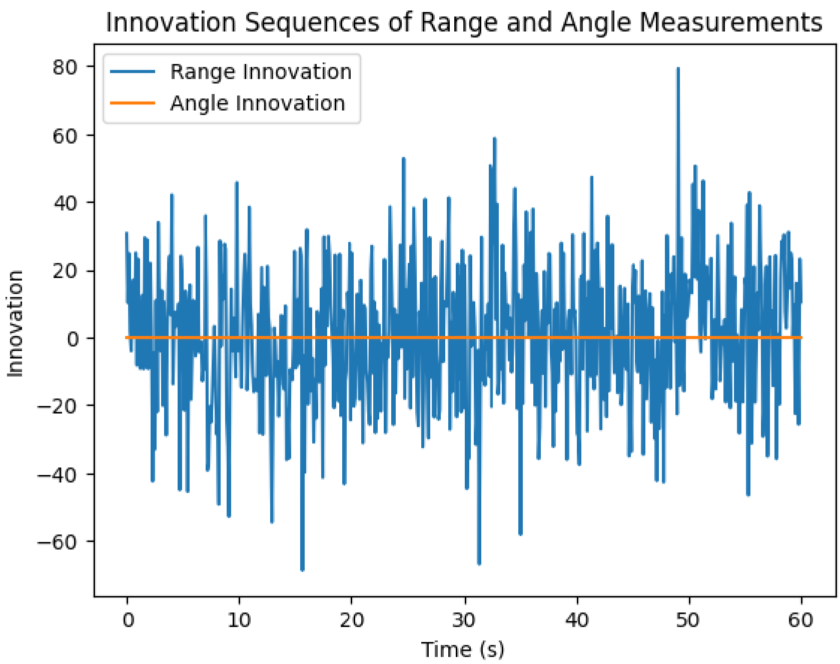

Figure 3: Innovation Sequences of Range and Angle Measurements

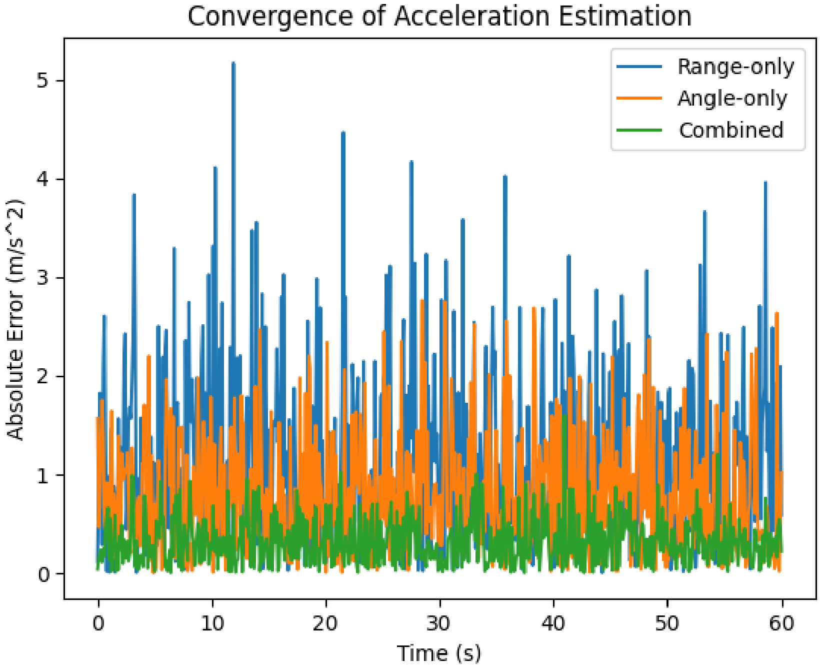

Figure 4: Convergence of Acceleration Estimation

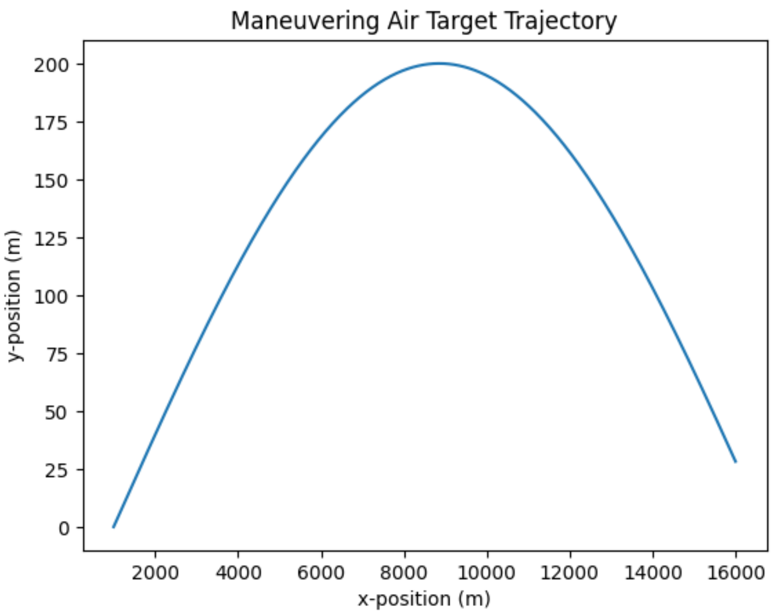

Figure 5. Maneuvering Air Target Trajectory

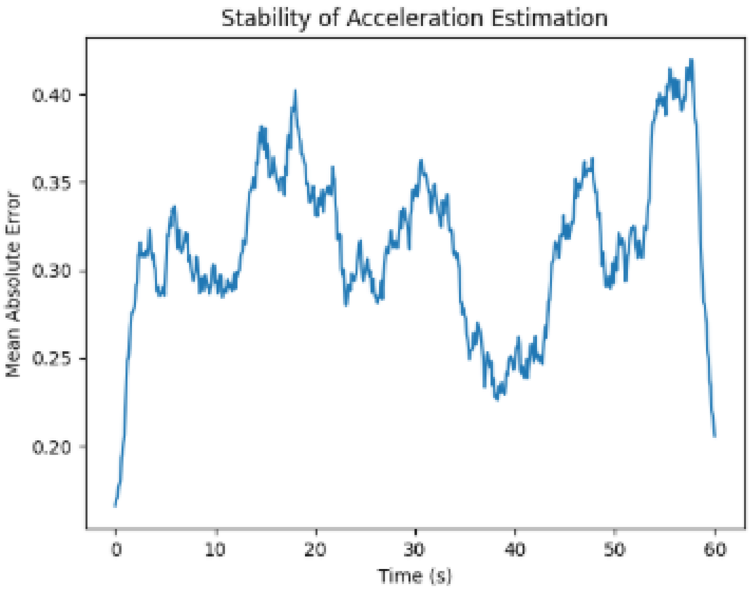

Figure 6. Stability of Acceleration Estimation

Simulation results demonstrate that the proposed combined filtering algorithm significantly outperforms single-channel approaches. During non-maneuvering phases, all filters exhibit comparable performance; however, clear differences emerge once the target begins maneuvering.

The range-only and angle-only filters show noticeable degradation in acceleration estimation accuracy during high-dynamic maneuvers, with increased estimation delay and oscillatory behavior. In contrast, the proposed combined filter maintains stable and accurate acceleration estimates, even during abrupt maneuver transitions.

The joint utilization of range and angular measurements enhances state observability and allows the filter to better capture rapid changes in target dynamics. As a result, the proposed algorithm achieves faster convergence and lower steady-state acceleration estimation errors compared to the reference methods.

These results confirm that combining range and angle information is essential for reliable acceleration estimation of maneuvering air targets, particularly in scenarios characterized by strong nonlinearities and measurement noise.

4. Conclusion

This paper has presented the synthesis of a combined range-measuring and angle-measuring channel filtering algorithm for estimating the acceleration of a maneuvering air target. By jointly processing nonlinear range and angular measurements within an augmented state-space framework, the proposed approach effectively enhances the observability of acceleration states and improves estimation accuracy under intensive target maneuvers.

The developed algorithm addresses the limitations of conventional single-channel filtering methods, which often suffer from performance degradation in the presence of strong target acceleration and measurement noise. Simulation results have demonstrated that the proposed combined filtering scheme provides faster convergence, reduced estimation errors, and improved robustness compared to range-only or angle-only approaches, particularly during abrupt and high-g maneuvers.

The proposed method is well suited for practical implementation in radar tracking systems, missile guidance, and air defense fire control applications, where accurate acceleration estimation is essential for trajectory prediction and interception performance. Future research will focus on extending the algorithm to three-dimensional target motion, incorporating adaptive maneuver detection mechanisms, and validating the approach using real radar measurement data.

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Declaration of Conflicting Interests

The authors declare that there are no conflicts of interest regarding the research, authorship, or publication of this article.

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