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Application of Decoupled Kalman Filtering for Angular State Estimation of Snake-Like Maneuvering Targets Under Noisy Environments

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Abstract

This paper presents a method applying a decoupled Kalman filter to estimate seven angular kinematic parameters relevant to missile guidance against a nonlinear maneuvering target in the presence of measurement noise. These parameters include: line-of-sight (LOS) angle, LOS rate, LOS angular acceleration, missile body angle, missile body angular rate, antenna angle, and antenna angular rate. The target model is constructed based on a highly nonlinear "snake-like" motion, reflecting strong maneuvering characteristics commonly seen in modern evasive tactics. The measured signals are synthesized by adding white Gaussian noise with specified variances for each variable. The Kalman filter system is designed in a decoupled structure consisting of three sub-filters. Filter performance is evaluated through quantitative metrics including Mean Square Error (MSE), Root Mean Square Error (RMSE), and Standard Deviation (STD) over time. Simulation results confirm the method's high accuracy and effectiveness under noisy measurements and strong target maneuvers, while also demonstrating computational advantages over the full Kalman filter.

Keywords: Decoupled Kalman Filter, Line-of-Sight, Missile Body Angle, Antenna Angle, State Estimation, Noisy Measurements, RMSE, MSE, STD.

1. Introduction

In modern radio guidance systems especially in air-to-air missiles or short-range defense missiles accurate determination of angular states is critical for effective control, tracking, and interception of targets. Parameters such as LOS angle, its rate and acceleration, as well as missile body and antenna angular positions and rates, play a vital role in control and precise navigation algorithms. However, in practice, sensor measurements are often affected by random noise, system bias, and hardware limitations. Furthermore, targets may perform complex nonlinear maneuvers such as "snake-like" motion, increasing the difficulty of state estimation problems. The Kalman filter is a powerful tool for state estimation under noisy measurements. However, for systems with many state variables and multi-layered kinematic structures like missile guidance, using a full Kalman filter may lead to high computational cost and impracticality for real-time hardware implementation. This paper proposes a decoupled Kalman filter structure consisting of three sub-filters, each handling a group of internally related angular state variables.

The estimated variables include:

- LOS angle j_n (deg), LOS angular rate ω_n (deg/s), LOS angular acceleration j_m (deg/s^2);
- Missile pitch angle J (deg), missile body angular rate ω_j (deg/s);
- Antenna angle j_a (deg), antenna angular rate ω_a (deg/s).

Each group of states is processed independently using time-invariant linear discrete-time state equations, with measurements corrupted by white Gaussian noise with predefined variance. The decoupling significantly reduces computational complexity and enhances real-time applicability in practical systems. Simulation results confirm the method's effectiveness and accuracy in estimating angular states under strong maneuvering and noisy measurement conditions.

2. Algorithm Formulation

To verify the effectiveness of the proposed decoupled Kalman filter in estimating angular states of a maneuvering target, a simulation model is constructed with key components: target kinematic model, noisy measurement model, decoupled Kalman filter system with three sub-filters, and an error evaluation process. The steps are described in detail as follows. The target moves in a “snake-like” trajectory characterized by time-varying normal acceleration, described by a harmonic function.

- Target kinematic model:

$$a_t(t) = a_{\max}\sin(\frac{2\pi t}{T_{mt}}) \tag{1}$$

Where:

$a_{\max}$ - is the maximum normal acceleration (m/s^2);

T_{mt} - is the target’s motion period (s).

- LOS angular rate:

$$\omega_n(t) = \frac{a_t(t)}{V} \tag{2}$$

V - is the target velocity (m/s).

- LOS angle as the time integral of ω_n:

$$j_n(t) = \int_0^t \omega_n(t)dt \tag{3}$$

- LOS angular acceleration as the first derivative of ω_n :

$$j_m(t) = \frac{d\omega_n(t)}{dt} \tag{4}$$

- Missile pitch angle:

$$J(t) = \alpha_1 j_n(t)\,;\, 0 < \alpha_1 < 1 \tag{5}$$

- Antenna angle:

$$j_a(t) = \alpha_2 j_n(t)\,;\; 0 < \alpha_2 < 1 \tag{6}$$

- Relative angle between antenna and missile body:

$$\varphi(t) = j_a(t) - J(t) \tag{7}$$

- Antenna angular rate:

$$\omega_a(t) = \alpha_2 \omega_n(t) \tag{8}$$

- Relative angular rate of antenna to missile body:

$$\dot{\varphi}(t) = \omega_a(t) - \omega_J(t) \tag{9}$$

In the missile guidance and antenna stabilization control model, accurate determination of angular kinematic parameters is crucial to ensure precise tracking and stability during target engagement. However, due to measurement noise and modeling errors, these parameters often cannot be directly measured with high accuracy. To address this issue, this paper employs a linear discrete-time Kalman filtering algorithm to accurately estimate system states from noisy measurements. The entire system is divided into three separate Kalman filters, each responsible for estimating a group of physically and control-related angular kinematic states.

Filter 1:

- State model:

$$\begin{aligned} j_n(k+1) &= j_n(k) + \Delta t.\omega_n(k) + \frac{1}{2}\Delta t^2.j_m(k) + \xi_{jn}(k) \\ \omega_n(k+1) &= \omega_n(k) + \Delta t.j_m(k) + \xi_{\omega n}(k) \\ j_m(k+1) &= j_m(k) + \xi_{jm}(k) \end{aligned} \tag{10}$$

Δt - is the discrete time step (s);

$\xi_{jn}, \xi_{\omega n}, \xi_{jm}$ - is the process noise, Gaussian with variances $q_{jn}, q_{\omega n}, q_{jm}$.

- State prediction:

$$\begin{aligned} \hat{j}_n^-(k+1) &= \hat{j}_n(k) + \Delta t.\hat{\omega}_n(k) + \frac{1}{2}\Delta t^2.\hat{j}_m(k) \\ \hat{\omega}_n^-(k+1) &= \hat{\omega}_n(k) + \Delta t.\hat{j}_m(k) \\ \hat{j}_m^-(k+1) &= \hat{j}_m(k) \end{aligned} \tag{11}$$

- Covariance prediction:

Assuming $P_1(k)$ is the 3x3 covariance matrix at time k and $Q_1 = diag(q_{jn}, q_{\omega n}, q_{jm})$, then:

$$P_1^-(k+1) = F_1 P_1(k) F_1^T + Q_1 \tag{12}$$

Where:

$$F_1 = \begin{bmatrix} 1 & \Delta t & 0.5\Delta t^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix}$$

- Kalman gain calculation:

Assuming measurement noise variance $R_1 = diag(r_{jn}, r_{\omega n}, r_{jm})$, then:

$$K_1(k+1) = P_1^-(k+1)H_1^T H_1 P_1^-(k+1)H_1^T + R_1^{-1}\;;\; H_1 = I_{3\times3} \tag{13}$$

- State update:

With measurement $z_1(k+1) = \left[z_{jn}, z_{\omega n}, z_{jm} \right]^T$, then:

$$\hat{x}_1(k+1) = \hat{x}_1^-(k+1) + K_1(k+1).z_1(k+1) - H_1\hat{x}_1^-(k+1) \tag{14}$$

- Covariance update:

$$P_1(k+1) = (I - K_1(k+1)H_1)P_1^-(k+1) \tag{15}$$

Filter 2:

- State model:

$$\begin{aligned} J(k+1) &= J(k) + \Delta t\omega_j(k) + \xi_J(k) \\ \omega_j(k+1) &= \omega_j(k) + \xi_{\omega_j}(k) \end{aligned} \tag{16}$$

- State prediction:

$$\begin{aligned} \hat{J}^-(k+1) &= \hat{J}(k) + \Delta t\hat{\omega}_j(k) \\ \hat{\omega}_j^-(k+1) &= \hat{\omega}_j(k) \end{aligned} \tag{17}$$

- Covariance prediction:

$$P_2^-(k+1) = F_2P_2(k)F_2^T + Q_2 \tag{18}$$

Where: $F_2 = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}; Q_2 = diag(q_J, q_{\omega j})$

- Kalman gain:

$$K_2(k+1) = P_2^-(k+1)H_2^T H_2 P_2^-(k+1)H_2^T + R_2^{-1} \tag{19}$$

$H_2 = I_{2\times 2}; R_2 = diag(r_J, r_{\omega j})$

- State update:

$$\hat{x}_2(k+1) = \hat{x}_2^-(k+1) + K_2(k+1)z_2(k+1) - \hat{x}_2^-(k+1) \tag{20}$$

- Covariance update:

$$P_2(k+1) = (I - K_2(k+1)H_2)P_2^-(k+1) \tag{21}$$

Filter 3:

- State model:

$$\begin{aligned} j_a(k+1) &= j_a(k) + \Delta t\omega_{ja}(k) + \xi_{ja}(k) \\ \omega_a(k+1) &= \omega_a(k) + \xi_{\omega_a}(k) \end{aligned} \tag{22}$$

- State prediction:

$$\begin{aligned} \hat{j}_a^-(k+1) &= \hat{j}_a(k) + \Delta t\hat{\omega}_a(k) \\ \hat{\omega}_a^-(k+1) &= \hat{\omega}_a(k) \end{aligned} \tag{23}$$

- Covariance prediction:

$$P_3^-(k+1) = F_3P_3(k)F_3^T + Q_3 \tag{24}$$

Where: $F_3 = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}; Q_3 = diag(q_{ja}, q_{\omega a})$

- Kalman gain:

$$K_3(k+1) = P_3^-(k+1)H_3^T H_3 P_3^-(k+1)H_3^T + R_3^{-1} \tag{25}$$

$$H_3 = I_{2\times 2}; R_3 = diag(r_{ja}, r_{\omega a})$$

- State update:

$$\hat{x}_3(k+1) = \hat{x}_3^-(k+1) + K_3(k+1)z_3(k+1) - \hat{x}_3^-(k+1) \tag{26}$$

- Covariance update:

$$P_3(k+1) = (I - K_3(k+1)H_3)P_3^-(k+1) \tag{27}$$

3. Simulation and Evaluation

To evaluate the effectiveness of the synthesized algorithm, the following simulation parameters are selected:

- Simulation duration: 50 (s); Sampling interval: 0.05 (s); Target velocity: 500 (m/s);
- Maximum normal acceleration of the target: 20 (m/s^2); Target maneuvering period: 10 (s);
- Angular parameters such as missile body angle and antenna angle are assumed equal $\alpha_1 = 0.5, \alpha_2 = 0.7$ to the LOS angle.

Kalman Filter 1 uses a process noise covariance matrix of 0.01, measurement noise of 0.25, and a measurement noise standard deviation of 0.5. Kalman Filters 2 and 3 use process noise covariance matrices of 0.01, measurement noise of 0.09, and measurement noise standard deviations of 0.3. Based on the synthesized algorithm and the selected simulation parameters, the following results are obtained:

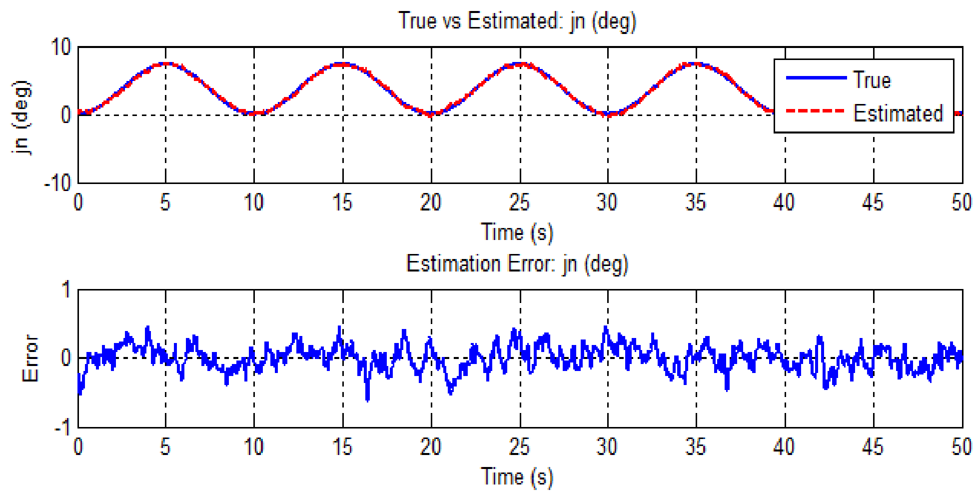

Figure 1. Comparison of true and estimated LOS angle j_n

Figure 1 shows the comparison between the true LOS angle j_n and the estimated value obtained from the decoupled Kalman filter. The estimated trajectory closely follows the true signal, demonstrating the filter's ability to accurately track the snake-like maneuvering motion of the target. The estimation error fluctuates slightly, mostly within ±0.5 degrees, with a standard deviation of approximately 0.3–0.5 degrees and an MSE of less than 0.1. This confirms the filter's capability in effectively suppressing noise and providing accurate estimation under conditions of continuous target movement and white Gaussian measurement noise.

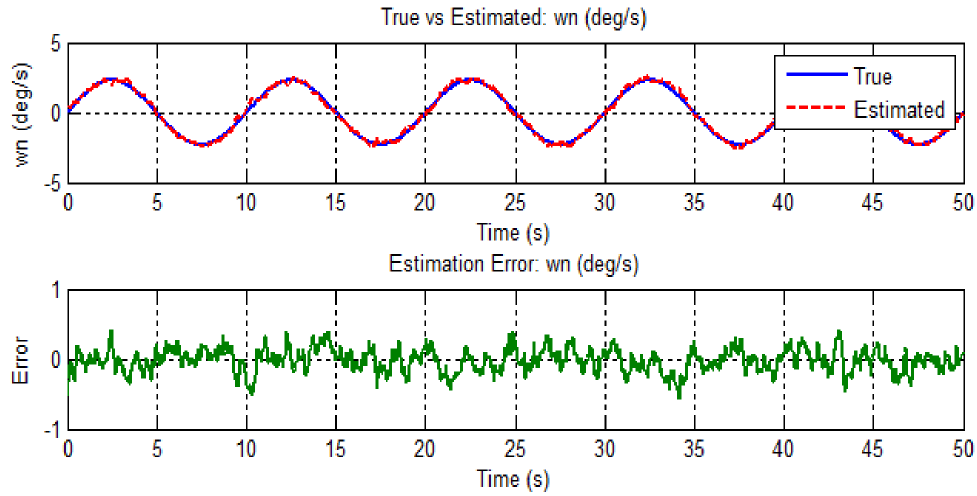

Figure 2. Comparison of true and estimated LOS angular rate ω_n

Figure 2 illustrates the estimated LOS angular rate ωn using the decoupled Kalman filter. The estimated values match the true values almost perfectly, with small and stable errors throughout the simulation duration. The errors are symmetrically distributed around zero, with amplitudes mostly under ±0.5 deg/s, standard deviation around 0.3–0.4 deg/s, and an MSE below 0.1. This result validates the filter's capability to accurately estimate LOS rate even under Gaussian measurement noise, ensuring robust tracking performance for highly maneuverable targets.

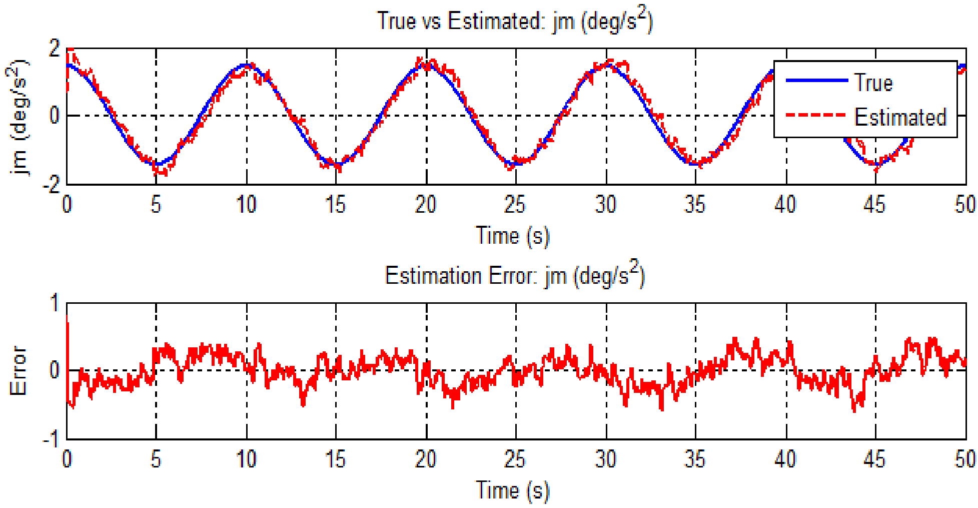

Figure 3. Comparison of true and estimated LOS angular acceleration j_m

Figure 3 shows the estimated LOS angular acceleration j_m obtained using the decoupled Kalman filter. The estimation curve (red dashed line) closely follows the true signal (blue solid line), accurately capturing the variation of LOS angular acceleration under measurement noise. The error mainly fluctuates within ±0.5 deg/s^2, with an estimated standard deviation around 0.3–0.5 deg/s^2 and an MSE under 0.1. Compared to previous angular parameters, the acceleration exhibits higher sensitivity to noise, yet the filter still maintains a low error, effectively supporting continuous tracking of maneuvering targets.

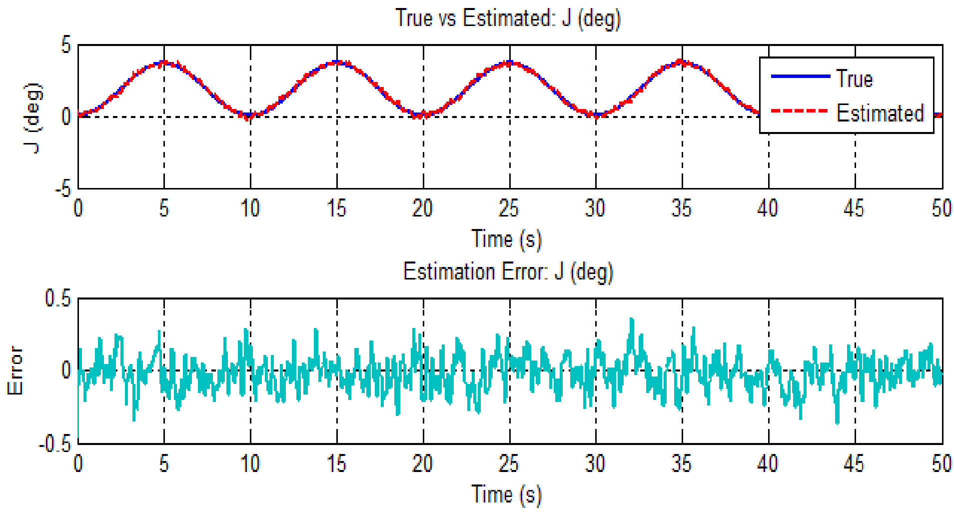

Figure 4. Comparison of true and estimated missile body angle J

Figure 4 presents the estimation of the missile body angle *J* using the decoupled Kalman filter. The estimated trajectory (red dashed) aligns almost exactly with the true signal (blue solid), with minimal and stable errors throughout the simulation. Errors are centered around zero with very low amplitude, mostly within ±0.3 degrees, standard deviation around 0.2–0.3 degrees, and MSE below 0.05. This confirms the filter's ability to accurately estimate the missile body angle under noisy measurements, supporting body stability control in guidance tasks.

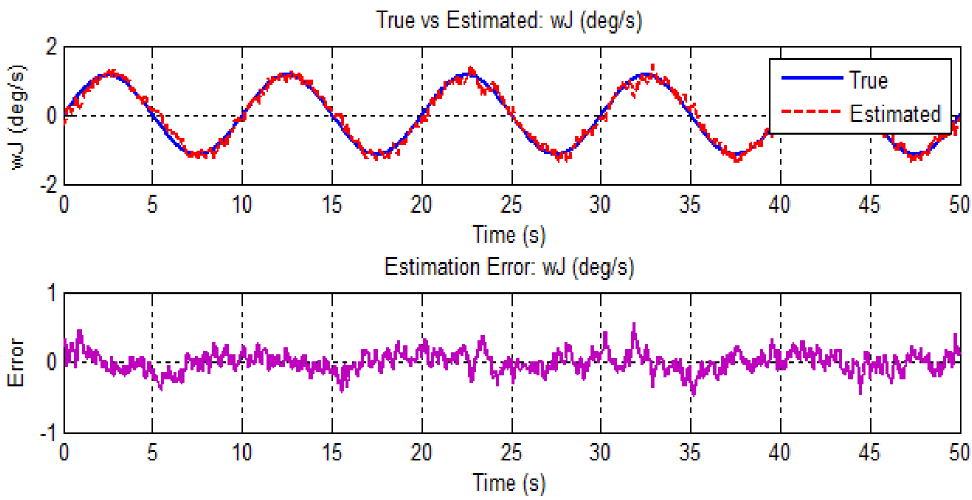

Figure 5. Comparison of true and estimated missile body angular rate ω_j

This figure shows the estimation of missile body angular rate ωj using the decoupled Kalman filter. The estimated curve closely matches the true signal, highlighting accurate tracking performance under noisy conditions. The error fluctuates symmetrically around zero, mostly within ±0.5 deg/s, standard deviation around 0.3–0.4 deg/s, and an MSE below 0.1. These results affirm the filter's accuracy and stability when estimating ω_j.

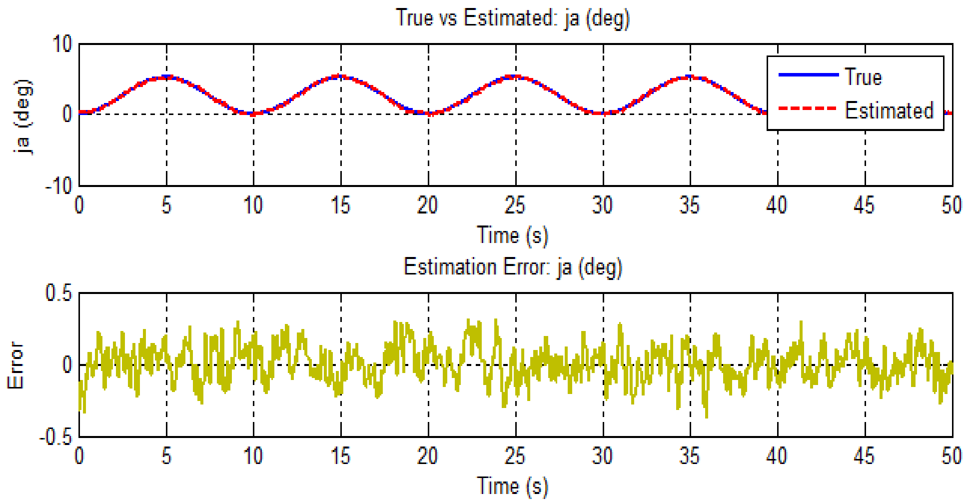

Figure 6. Comparison of true and estimated antenna angle j_a

The estimated antenna angle *ja* closely follows the true signal, indicating high accuracy throughout the simulation. The error is small, symmetrically distributed around zero, with amplitudes not exceeding ±0.3 degrees, standard deviation around 0.2–0.3 degrees, and MSE below 0.05. These results demonstrate that the filter effectively processes noisy measurements in estimating antenna angle, meeting the accuracy and stability requirements for antenna control in guidance systems.

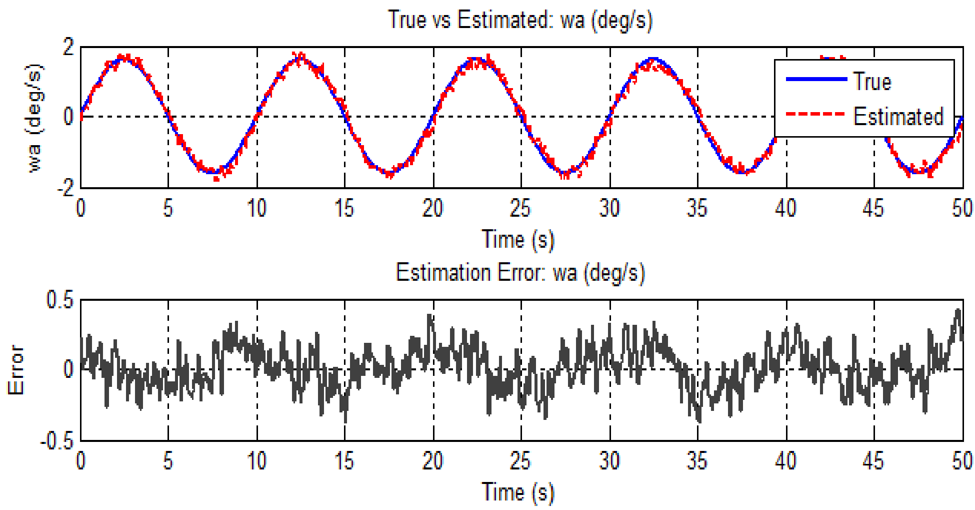

Figure 7. Comparison of true and estimated antenna angular rate ω_a

Figure 7 shows the estimated antenna angular rate ω_a using the decoupled Kalman filter. The estimation curve (red dashed) closely follows the true signal (blue solid), clearly reflecting the filter's strong tracking capability under noisy measurements.

The error is small, fluctuating around zero, with amplitudes mostly below ±0.3 deg/s, standard deviation around 0.2–0.3 deg/s, and MSE under 0.05. This result confirms the filter's high precision in estimating antenna angular rate, ensuring stability and accuracy in the guidance control system.

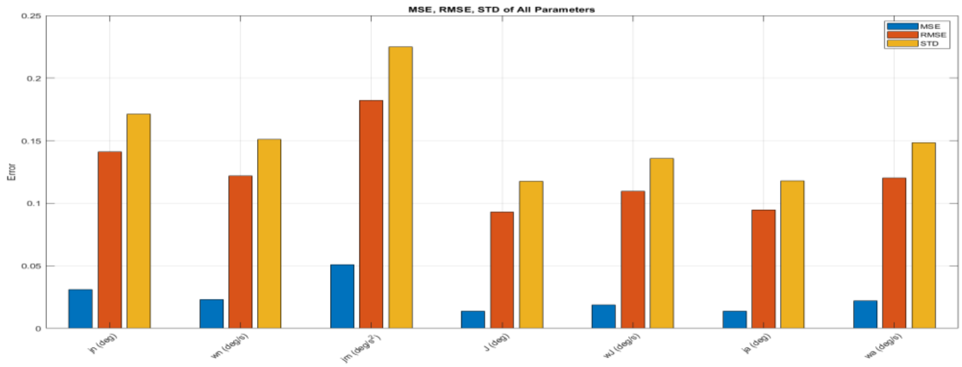

Figure 8. Evaluation of MSE, RMSE and STD for all angular parameters

Figure 8 summarizes the MSE, RMSE and STD results for the seven estimated angular parameters. All MSE values are below 0.05, reflecting very high accuracy. RMSE values range from 0.1 to 0.17, with the highest for *jm* (LOS angular acceleration) at approximately 0.17. STD values range from 0.12 to 0.2, also highest for j_m. The missile body (J, ω_j) and antenna (j_a, ω_a) parameters show smaller errors compared to the LOS-related parameters due to lower measurement noise. These results confirm that the decoupled Kalman filter performs effectively across all parameters, maintaining low and stable errors under measurement noise.

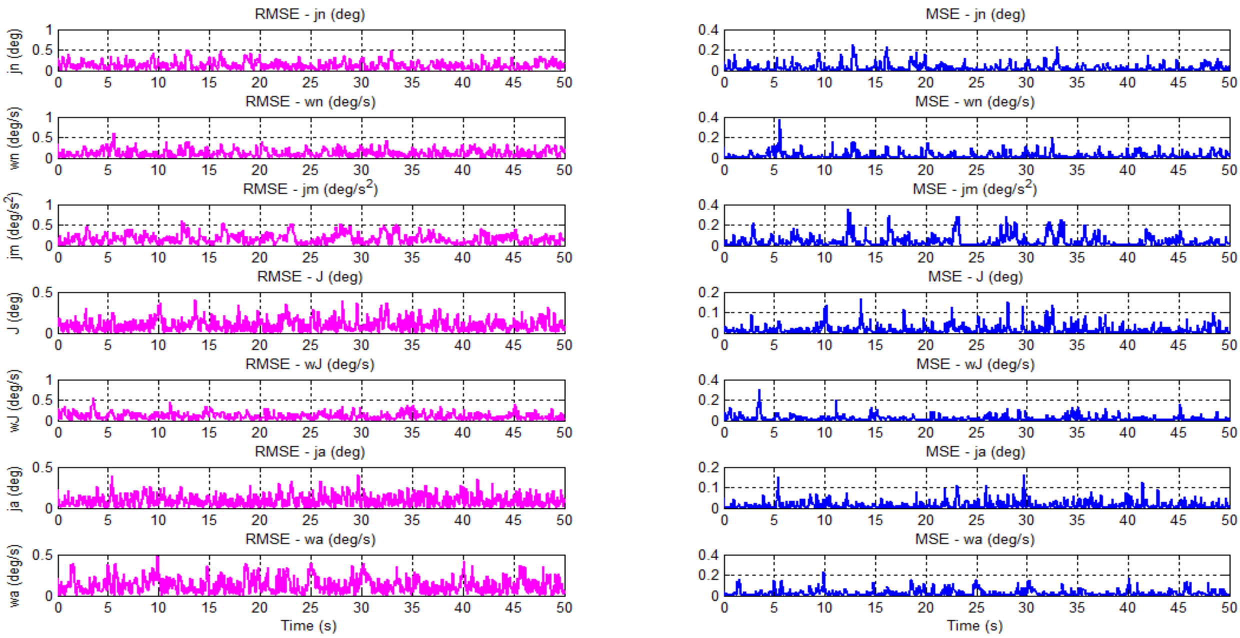

Figure 9. Evaluation of RMSE and MSE of angular parameters over time

Figure 9 presents the temporal evolution of RMSE (left) and MSE (right) for the seven estimated angular parameters. RMSE values remain consistently low, mostly under 0.2. The parameter j_m (LOS angular acceleration) has the largest fluctuation, but stays below 0.5. Other parameters maintain RMSE below 0.3, indicating that the filter preserves good accuracy throughout the 50-second simulation. Meanwhile, MSE values fluctuate within a narrower range, below 0.1, with peaks occurring during periods of intense target maneuvering or increased measurement noise. These results demonstrate the stability and effectiveness of the decoupled Kalman filter under noisy conditions, even for highly sensitive parameters like LOS angular rate and acceleration.

4. Conclusion

This paper presented a method of applying a decoupled Kalman filter to estimate seven angular kinematic parameters in an air-to-air missile guidance system while tracking a snake-like maneuvering target under measurement noise. Simulation results show that the filter achieves high accuracy, with Mean Square Error (MSE) values all below 0.1 and Standard Deviation (STD) values mostly under 0.2 degrees or degrees per second. Even highly noise-sensitive parameters such as LOS angular rate and acceleration were accurately estimated, with error fluctuations remaining stable and symmetrically distributed around zero. The use of a decoupled Kalman filter structure significantly reduces computational complexity, making it suitable for real-time requirements in modern guidance systems. The proposed method demonstrates effectiveness in environments with noisy measurements and strong target maneuvers, and serves as a feasible approach to enhance estimation accuracy while optimizing hardware cost in real-world nonlinear target tracking problems.

Declaration of Conflicting Interests

The authors declare no potential conflicts of interest with respect to the research, authorship and publication of this article.

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