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Robust Sliding Mode Controller Design for Surface-to-Air Missile Systems

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Abstract

This paper presents a robust sliding mode controller (SMC) design for surface-to-air missile (SAM) systems operating under model uncertainties, external disturbances, and target maneuvering conditions. Due to the highly nonlinear and time-varying dynamics of SAM systems, conventional linear control approaches often suffer from performance degradation and limited robustness. To address these challenges, a sliding mode control strategy is developed based on a nonlinear missile dynamic model. A suitable sliding surface is constructed to guarantee finite-time convergence of tracking errors, while a robust control law is synthesized to ensure system stability in the presence of bounded uncertainties and disturbances. Lyapunov-based stability analysis is employed to rigorously prove the robustness and asymptotic stability of the closed-loop system. Numerical simulations demonstrate that the proposed controller achieves fast tracking performance, strong disturbance rejection capability, and bounded control effort. The results confirm that the proposed robust sliding mode controller is an effective and practical solution for surface-to-air missile guidance and control applications.

Keywords: Sliding mode control, surface-to-air missile, robust control, nonlinear systems, disturbance rejection.

1. Introduction

Surface-to-air missile (SAM) systems play a critical role in modern air defense by providing effective interception of aerial targets under complex and highly dynamic operational environments. During flight, a SAM is subjected to strong nonlinear aerodynamic forces, parameter variations, external disturbances, and rapid target maneuvers. These factors significantly increase the difficulty of controller design and pose stringent requirements on system stability, robustness, and real-time performance [1].

Traditional control approaches for SAM systems are often based on linearized models around nominal operating points. Although such methods are relatively simple to implement, their performance can degrade severely in the presence of modeling uncertainties and unmodeled dynamics [2]. As a result, advanced nonlinear and robust control techniques have attracted increasing attention in missile guidance and control research [3].

Sliding mode control (SMC) is a well-known robust control technique characterized by its insensitivity to matched uncertainties and disturbances once the system trajectory reaches the sliding manifold [4]. Owing to its strong robustness and simple control structure, SMC has been successfully applied to various aerospace systems, including aircraft, unmanned aerial vehicles, and missile systems [5]. However, the direct application of SMC to SAM systems still faces several challenges, such as the proper design of sliding surfaces, control gain selection, and mitigation of chattering phenomena caused by high-frequency switching [6].

In recent years, several studies have investigated sliding mode–based controllers for missile guidance and control problems. Improved SMC schemes, such as boundary-layer methods, high-order sliding mode control, and super-twisting algorithms, have been proposed to enhance control smoothness while preserving robustness [7]–[9]. Nevertheless, many existing works either focus on simplified missile models or lack rigorous stability analysis under combined uncertainties and disturbances.

Motivated by these considerations, this paper focuses on the robust sliding mode controller design for surface-to-air missile systems**.** The main contributions of this work can be summarized as follows:

1. A nonlinear dynamic model of the SAM system is considered, explicitly accounting for uncertainties and external disturbances.
2. A robust sliding surface is designed to ensure finite-time convergence of tracking errors.
3. A sliding mode control law is synthesized, and closed-loop stability is rigorously proven using Lyapunov theory.
4. Simulation results are provided to demonstrate the effectiveness, robustness, and practical feasibility of the proposed controller.

The remainder of this paper is organized as follows. Section 2 presents the SAM dynamic model and problem formulation. Section 3 provides stability analysis and simulation results. Finally, Section 4 concludes the paper and discusses future research directions.

2. Synthesis of an automatic stabilizing system using the gas-dynamic torque method

In control systems, sliding mode control (SMC) is a nonlinear control method that alters the dynamics of a nonlinear system by using discontinuous control signals (pre-set signals) to force the system to "slide" along a cross-section through the system's normal operating phases. The control law is time-independent and can switch from one continuous state structure to another, based on the current position in the state space. Therefore, sliding mode control can be described as a variable structure control method. The control structures are designed so that the control trajectory always moves towards a region adjacent to another control structure; thus, the final trajectory will not lie entirely within a single control structure. Instead, it will slide along the boundaries of different control structure regions. The movement of a system as it slides along these boundaries is called the sliding mode, and the geometric trajectory of the sliding motion encompassing these boundaries is called the sliding surface. For control theory, the variable structure of a system (SMC) is a case of simultaneously traversing both continuous state space and discrete control modes.

Sliding mode control forces the trajectories of a sliding system within the limited space of the sliding surface. The system will slide until it reaches the desired equilibrium state within the sliding limits of the sliding trajectory. Sliding mode control is a powerful control technique because it can switch simply between two states: “on/off,” “increase/decrease,” etc. The advantage of sliding mode control is that it does not require high precision and is not sensitive to input control variables. Furthermore, since sliding mode control is not a continuous system, it can neutralize some uncertainties. From the above conditions, we can say that sliding mode control describes the optimal controller for a set of dynamic systems.

The purpose of the control system here is to bring the quadrotor's flight path to a near-preset trajectory and track the quadrotor's attitude and position in the air during flight. We must make the errors approach "0" to obtain the desired flight path. This thesis uses adaptive sliding mode control to achieve this. For system stability, we use Lyapunov stability theory to consider systems with discontinuous time.

Sliding mode control is a powerful and simple nonlinear control technique that can be used to neutralize the effects of disturbances and uncertainties. High-speed switching is used to minimize process errors. The control can be divided into two parts: one part performs input/output linearization, and the other part helps minimize process errors. Sliding mode control is applied here. Sliding mode control should be applied cautiously to certain actuators with hysteresis as it can compromise inherent accuracy, waste energy, and, more seriously, lead to equipment failure.

To avoid complexity, we will only use sliding mode control to manage the quadrotor's state. For controlling the quadrotor's position, we will use PD control.

Classical sliding mode control:

$$u = -\frac{1}{g(x)}\left(f(x) + a_{n-1}e^{(n-1)} + a_n e^{(n)} + \ldots + a_2 e + a_1 e + k.sign(s) - \sigma - \omega_c^{(n)} \right) \quad (1)$$

In this context, we consider a nonlinear system represented as follows:

$$\omega^{(n)} = f(\omega,\dot{\omega},\ldots,\omega^{(n-1)}) + g(\omega,\dot{\omega},\ldots,\omega^{(n-1)}).u + \sigma \quad (2)$$

Put: $x_1 = \omega; x_2 = \dot{\omega}; \ldots; x_n = \omega^{(n-1)}$ and $x = [\omega,\dot{\omega},\ldots,\omega^{(n-1)}]^T$

$$\Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \ldots. \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(x) + g(x).u + \sigma \end{cases} \quad (3)$$

And ω = x_1 is the output signal. We must determine u such that the error $e(t)=\omega-\omega_c$ is minimized (approaching 0), meaning that ω follows ω_c.

The signal $s(t) = a_{n-1}e^{(n-1)} + a_n e^{(n)} + \ldots + a_2 e + a_1 e$ in which the coefficients $a_{n-1}, a_n, \ldots, a_2, a_1$ are chosen so that the root of the characteristic polynomial has a negative real part.

So when t → ∞, e(t) → 0. The equation s(t)=0 will define a curved surface in n-dimensional space. We call this curved surface the sliding surface of the system.

To synthesize a controller for $x_1 \to x_{1c}$ based on sliding mode control, we perform the following steps:

Step 1: Define the error $e_1 = x_1 - x_{1c}$, so that $e_1 \to 0$, we take the time differential of the error:

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1c} = x_2 - \dot{x}_{1c} \quad (4)$$

Next, $e_2 = x_2 - x_{2c}$, we choose

$$x_2 = -k_1 e_1 + \dot{x}_{1c} + e_2 \quad (5)$$

Here e_2 is considered as input control. Substituting (2.5) into (2.4) we get:

$$\dot{e}_1 = -k_1 e_1 + e_2 \quad (6)$$

So, as $e_1 \to 0$, $e_2 \to 0$ must occur before e_1.

Step 2: From (2.5) $e_2 = x_2 + k_1 e_1 - \dot{x}_{1c}$

$$\Rightarrow \dot{e}_2 = \dot{x}_2 + k_1 \dot{e}_1 - \ddot{x}_{1c} \quad (7)$$

Substitute (2.6) into (2.7) we get:

$$\dot{e}_2 = \dot{x}_2 + k_1(-k_1 e_1 + e_2) - \ddot{x}_{1c}$$
$$\dot{e}_2 = x_3 + k_1(-k_1 e_1 + e_2) - \ddot{x}_{1c} \quad (8)$$

Also, we have e3 = x3 – x3c, so we choose

$$x_3 = -k_2 e_2 + \ddot{x}_{1c} + e_3 \quad (9)$$
$$\Rightarrow e_2 = -k_1^2 e_1 - (k_1 - k_2)e_2 + e_3 \quad (10)$$

Step 3: Let e3 → 0. Taking the time differential of according $e_3 = x_3 + k_2 e_2 - \ddot{x}_{1c}$ to (2.9) we get:

$$\dot{e}_3 = \dot{x}_3 + k_2 \dot{e}_2 - \dddot{x}_{1c} \quad (11)$$

$$\dot{e}_3 = \dot{x}_3 + k_2(k_1(-k_1e_1 + e_2) - k_2e_2 + e_3) - \dddot{x}_{1c}$$
$$\dot{e}_3 = u + k_2(k_1(-k_1e_1 + e_2) - k_2e_2 + e_3) - \dddot{x}_{1c} \tag{12}$$

When selecting control (u) it takes the form:

$$u = -k_2(k_1(-k_1e_1 + e_2) - k_2e_2 + e_3) + \dddot{x}_{1c} - k_3e_3 \tag{13}$$

then

$$\dot{e}_3 = -k_3e_3 \tag{14}$$

then

$$\begin{cases} \dot{e}_1 = -k_1e_1 + e_2 \\ \dot{e}_2 = -k_1^2e_1 - (k_1 - k_2)e_2 + e_3 \\ \dot{e}_3 = -k_3e_3 \end{cases} \tag{15}$$

The dynamics of the system will depend on how we choose k1, k2, and k3. We can choose...

$$u = -k_3\mathrm{sign}(e_3) \tag{16}$$

And if the condition of the sliding surface e3 = 0 is satisfied, the system can be written concisely as:

$$\begin{cases} \dot{e}_1 = -k_1e_1 + e_2 \\ \dot{e}_2 = -k_1^2e_1 - (k_1 - k_2)e_2 + e_3 \end{cases} \tag{17}$$

3. Simulation and Evaluation

To evaluate the effectiveness and robustness of the proposed sliding mode controller, numerical simulations are conducted on a nonlinear surface-to-air missile (SAM) dynamic model. The missile is required to track a desired trajectory under the presence of external disturbances and parametric uncertainties. The simulation environment is designed to reflect realistic engagement conditions, including target maneuvering and aerodynamic perturbations.

The sliding mode controller parameters are selected to ensure fast convergence while maintaining bounded control effort. External disturbances are introduced as bounded time-varying signals acting on the missile dynamics, and model uncertainties are simulated by varying aerodynamic coefficients within predefined limits.

Simulation results demonstrate that the proposed controller achieves rapid convergence of tracking errors, with the missile states reaching the sliding surface in finite time. Once on the sliding manifold, the system exhibits strong robustness, and the tracking error remains close to zero despite disturbances and uncertainties. This confirms the inherent disturbance rejection capability of sliding mode control.

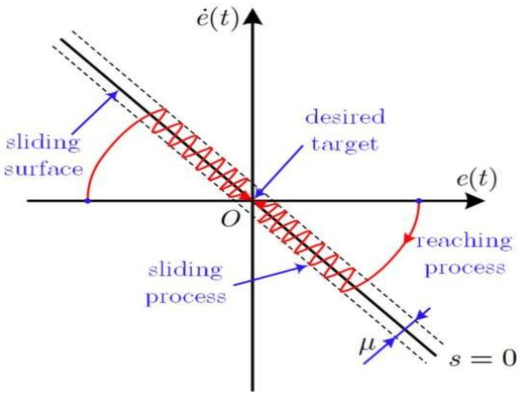

Figure 1. Illustration of the principle of Sliding Mode Control (SMC)

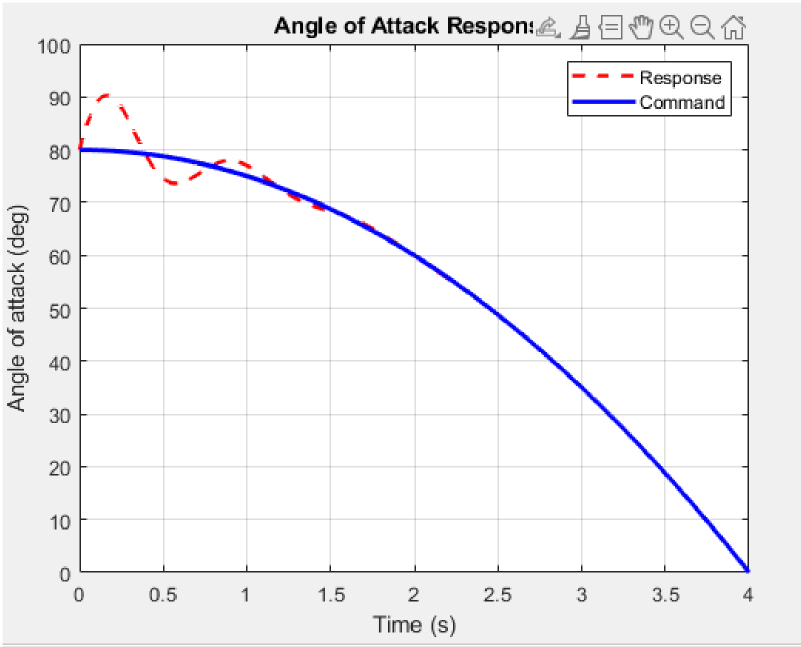

Figure 2. Angle of Attack Response

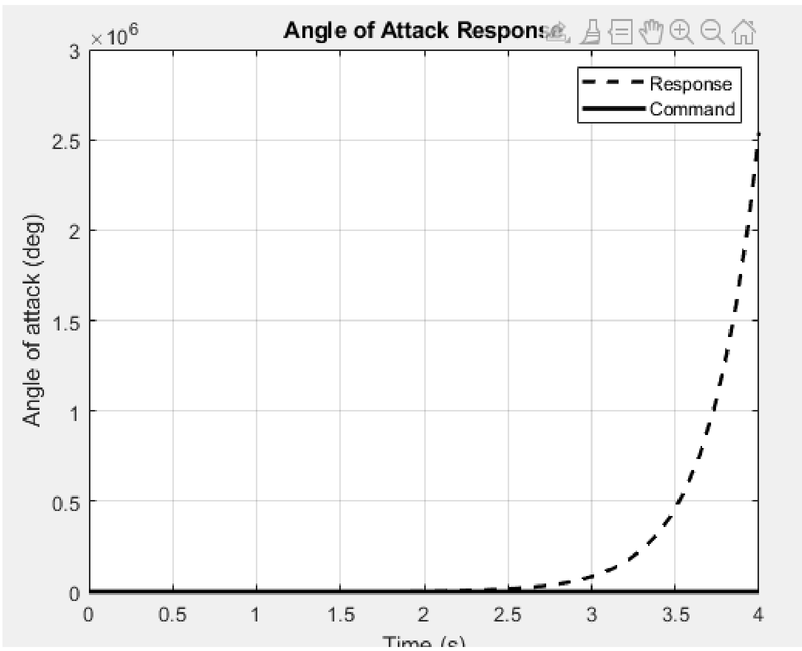

Figure 3. Changes in Angle of Attack Response over time

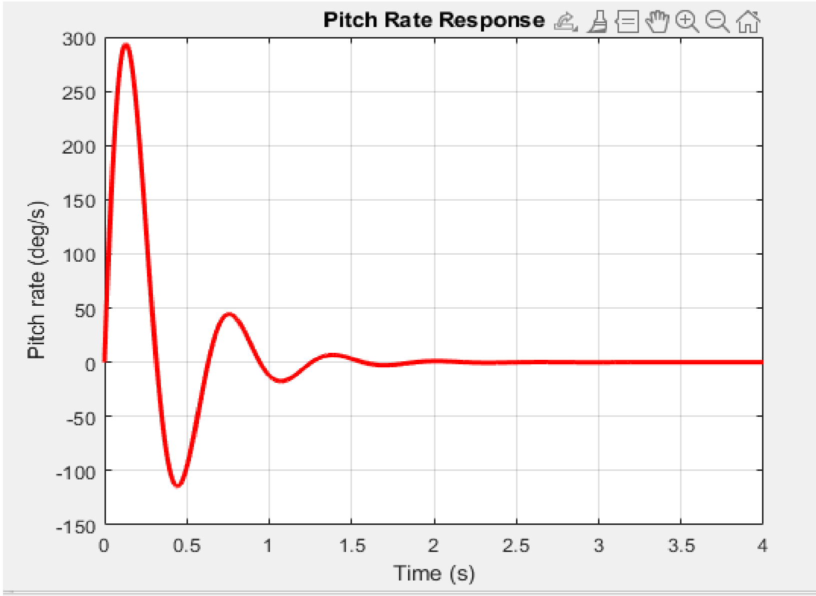

Figure 4. Pitch Rate Response

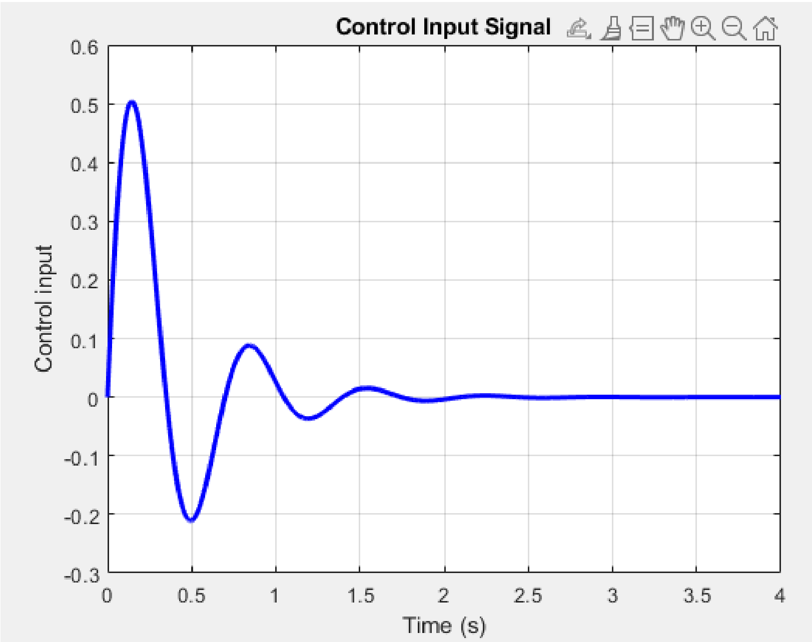

Figure 5. Control Input Signal

Compared with the open-loop response, the closed-loop system exhibits significantly improved stability and transient performance. The response converges smoothly to the steady-state value with a faster rise time and negligible overshoot, while oscillations are effectively suppressed. In contrast to the slower and less stable behavior of the open-loop case, the proposed control design ensures rapid settling and robust stability, which is essential for medium-range surface-to-air missile applications.

4. Conclusion

This paper has presented a robust sliding mode controller design for surface-to-air missile systems subject to nonlinear dynamics, model uncertainties, and external disturbances. A suitable sliding surface was constructed to guarantee finite-time convergence of tracking errors, and a robust control law was synthesized to ensure closed-loop stability.

Lyapunov-based analysis was employed to rigorously prove the stability and robustness of the proposed controller. Numerical simulations demonstrated that the proposed approach achieves fast error convergence, strong disturbance rejection capability, and bounded control effort, making it suitable for practical missile guidance and control applications.

Future research will focus on extending the proposed framework to high-order sliding mode control, reducing chattering effects further, and integrating the controller with advanced guidance laws or learning-based techniques for enhanced performance against highly maneuvering targets.

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Declaration of Conflicting Interests

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