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Optimal Guidance Law Design for Air-to-Air Missiles Intercepting Slow and Highly Maneuverable Targets

Nguyen Ngoc Thien[1*]

[1]*Missile Faculty, Air Defence-Air Force Academy, Ha Noi, Viet Nam*

**Corresponding author: anhbodoj838@gmail.com*

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Abstract

This paper presents the design of an optimal guidance law for air-to-air missiles (AAMs) engaging slow and highly maneuverable aerial targets, such as helicopters or unmanned aerial vehicles. Unlike conventional proportional navigation (PN), which is optimized for high-speed targets, the proposed approach explicitly accounts for target maneuverability and low-speed kinematic characteristics. An optimal control formulation is established to minimize miss distance while constraining missile acceleration. The resulting guidance law is derived analytically and implemented in a closed-loop form. Numerical simulations demonstrate that the proposed method significantly improves interception performance compared with classical PN under aggressive target maneuvers.

Keywords: Air-to-air missile, optimal guidance, slow maneuvering targets, proportional navigation, optimal control..

1. Introduction

Air-to-air missile (AAM) guidance law design has long been a central research topic in missile guidance and control theory. Among various guidance strategies, proportional navigation (PN) and its variants have been widely adopted in operational missile systems due to their simplicity, robustness, and proven effectiveness against high-speed, weakly maneuvering aerial targets such as fighter aircraft and cruise missiles [1], [2]. Classical PN guidance relies on the fundamental assumption that the target velocity is significantly higher than the missile's lateral maneuver capability and that target acceleration can be neglected or treated as a bounded disturbance.

However, the modern aerial combat environment has changed significantly in recent decades. Air-to-air missiles are increasingly required to engage slow and highly maneuverable targets, including helicopters, rotary-wing aircraft, and low-speed unmanned aerial vehicles (UAVs). These targets exhibit unique kinematic and dynamic characteristics that differ substantially from those of conventional high-speed aircraft. In particular, helicopters are capable of sharp lateral maneuvers, hovering, sudden velocity changes, and rapid heading reversals, which seriously challenge the effectiveness of classical PN-based guidance laws [3], [4]. For slow targets, the relative closing velocity between the missile and the target is reduced, leading to prolonged engagement time and increased sensitivity to line-of-sight (LOS) rate variations. Under such conditions, PN guidance often produces excessive lateral acceleration commands and oscillatory missile trajectories, which may result in increased miss distance or even interception failure when actuator saturation or acceleration constraints are present [5]. Moreover, the assumption of negligible target maneuver becomes invalid, and unmodeled target acceleration can significantly degrade guidance performance.

To address these limitations, numerous studies have proposed enhanced guidance laws incorporating target maneuver estimation or robustness mechanisms. Augmented proportional navigation (APN), for example, introduces explicit target acceleration compensation to improve interception accuracy against maneuvering targets [6]. Differential game–based guidance laws have also been investigated to account for adversarial target maneuvers [7]. While these approaches improve robustness, they often rely on accurate target acceleration estimation or require complex real-time computation, which may be impractical in highly uncertain combat scenarios.

Optimal control theory provides a systematic framework for guidance law synthesis by explicitly formulating missile interception as an optimization problem. By minimizing a suitably defined cost function—typically involving terminal miss distance and control effort—optimal guidance laws can be derived analytically or numerically [8]. Early optimal guidance formulations have demonstrated that PN can be interpreted as a special case of optimal guidance under simplified assumptions [9]. Nevertheless, many existing optimal guidance laws remain tailored to engagements involving high-speed targets and do not fully exploit the kinematic characteristics of slow and agile targets.

In recent years, there has been growing interest in developing guidance laws specifically designed for low-speed, highly maneuverable targets. Research efforts have focused on adaptive PN gains [10], sliding-mode guidance [11], and learning-based approaches [12]. Despite these advances, there remains a need for analytically derived optimal guidance laws that explicitly consider the interception of slow and highly maneuverable targets while maintaining implementation simplicity and robustness.

Motivated by these challenges, this paper presents an optimal guidance law for air-to-air missiles engaging slow and highly maneuverable targets. The proposed approach formulates the interception problem within an optimal control framework, leading to a closed-loop guidance law that naturally extends classical PN by incorporating optimal gain selection and target maneuver compensation. The resulting guidance law retains a simple structure suitable for real-time implementation while significantly improving interception performance under aggressive target maneuvers.

2. Synthesis of optimal guidance methods

For the general situation, the optimal control law synthesis problem uses linear models defined in state space, which can be expressed by the following differential equation:

$$\dot{\mathbf{x}}(t)=\mathbf{F}_{\mathbf{y}}(t)\mathbf{x}_{\mathbf{y}}(t)+\mathbf{B}_{\mathbf{y}}(t)\mathbf{u}(t)+\xi_x(t) \tag{1}$$

Intended for the development of the process:

$$\dot{\mathbf{x}}_T(t)=\mathbf{F}_T(t)\mathbf{x}_T(t)+\xi_T(t) \tag{2}$$

If measurements are available:

$$\mathbf{z}(t)=\mathbf{H}(t)\mathbf{x}(t)+\boldsymbol{\xi}_H(t) \tag{3}$$

Find the control law:

$$\mathbf{u}(t)=\mathbf{K}^{-1}\mathbf{B}_{\mathbf{y}}^{\mathbf{T}}\mathbf{Q}\left[\mathbf{x}_T(t)-\mathbf{x}_Y(t)\right] \tag{4}$$

Optimal for the minimum of local quality functionality:

$$\mathbf{I}=\mathbf{M}\left\{\left[\mathbf{x}_T(t)-\mathbf{x}_Y(t)\right]^T\mathbf{Q}\left[\mathbf{x}_T(t)-\mathbf{x}_Y(t)\right]+\int_0^t\mathbf{u}^T(t)\mathbf{K}\mathbf{u}(t)dt\right\} \tag{5}$$

where $\mathbf{F}_y(t)$ is the dynamic matrix of the process state;
$\mathbf{x}_y(t)$ is the n-dimensional vector of controlled phase coordinates;
$\mathbf{B}_y(t)$ is the matrix of control efficiency;
$\mathbf{u}(t)$ is the r-dimensional vector of control signals ($r \le n$);
$\xi_y(t)$ is the n-dimensional vector of centered Gaussian disturbances of the process
Matrix $\mathbf{G}_y$ of one-sided spectral densities;
$\mathbf{F}_T(t)$ is the dynamic matrix of the process state;
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Matrix $\mathbf{G}_T$ of one-sided spectral densities;

$\mathbf{H}(t)$ is the matrix of the connection of the generalized state vector

$\xi_H(t)$ – n-dimensional vector of centered Gaussian measurement noise with known matrix Gи of one-sided spectral densities;

K – matrix of penalties for the magnitude of control signals;– matrix that determines the ability of the system to perceive control signals;

Q – matrix of penalties for control accuracy;

M – mathematical expectation, provided that there are measurement results of at least part of the phase coordinates $\mathbf{x}_y$ and x_t.

The helicopter flight speed V_h can be represented as:

$$V_h = V_л k - V_\phi \cos\varphi_\phi \tag{6}$$

where k is the coefficient compensating for the "erroneous" acceptance of the blade speed as the speed of the helicopter as a whole, determined by the formula:

$$k = \frac{1}{V_h} = \frac{2}{\left(F_{dm} - F_{dh}\right)\lambda} \tag{7}$$

where F_{dm} is the Doppler frequency due to the proper speed of the guided missile;

F_{dh} is the Doppler frequency due to the proper linear speed of rotation of the advancing blade of the main rotor of the guided missile helicopter;

λ is the wavelength.

Projecting the velocities V_m and V_h onto the line of sight and the normal to it, we obtain:

$$\dot{D} = -V_л - V_m \cos\varphi_m \tag{8}$$

Substituting (6) into (8), we obtain:

$$D\omega = V_л - V_\phi \sin\varphi_\phi + V_m \sin\varphi_m \tag{9}$$

For simplicity, we assume that the cosines of all angles are approximately equal to unity, and the sines are equal to their arguments.

Differentiating (9) with respect to time at V_m = const, we obtain:

$$\dot{D}\omega + D\dot{\omega} = \dot{V}_л - \dot{V}_\phi \varphi_\phi - V_\phi \dot{\varphi}_\phi + V_m \dot{\varphi}_m \tag{10}$$

Or:

$$\dot{\omega} = \frac{\dot{V}_л - \dot{V}_\phi \varphi_\phi}{D} - \frac{V_\phi \dot{\varphi}_\phi - V_m \dot{\varphi}_m}{D} - \frac{\dot{D}\omega}{D} \tag{11}$$

Since: $V_m \dot{\varphi}_m = J_p; V_\phi \dot{\varphi}_\phi = J_\phi; \dot{V}_л - \dot{V}_\phi \varphi_\phi = W_h$ expression (11) can be written as

$$\dot{\omega} = \frac{1}{D}\left(W_p + W_p - J_\phi\right) - \frac{\dot{D}}{D}\omega \tag{12}$$

In the case when Vϕ≈0 (hover mode):

$$\dot{\omega} = \frac{1}{D}\left(W_p + W_h\right) - \frac{\dot{D}}{D}\omega \tag{13}$$

To determine the change in the helicopter's onboard bearing during the process of guiding the helicopter to it, we represent formula (9) as:

$$\varphi_m = \arcsin\left(\frac{D\omega - V_\pi + V_\phi \sin\phi}{V_m}\right) \tag{14}$$

Differentiating (14) with respect to time at V_m = const, we obtain:

$$\dot{\varphi} = \omega + \frac{1}{\dot{D}}\left(W_m + W_h - J_\phi\right) \tag{15}$$

Thus, based on formulas (12) and (15), the dynamic model describing the relative movement of the missile and the helicopter with different flight patterns has the following form.

$$\begin{cases} \dot{\varphi} = \omega + \dfrac{1}{\dot{D}}\left(W_m + W_h - J_\phi\right);\ \varphi(0) = \varphi_0 \\ \dot{\omega} = \dfrac{1}{D}\left(W_m + W_p - J_\phi\right) - \dfrac{\dot{D}}{D}\omega;\ \omega(0) = \omega_0 \end{cases} \tag{16}$$

This dynamic model of the relative movement of the missile and the helicopter will be further used as the basis for synthesizing the optimal method and algorithm for guiding the missile to the helicopter.

To synthesize the missile control law, it is necessary to have a state model (1), (2) and a quality functional in the form of (5).

Based on the fact that the control parameter is the lateral acceleration of the missile, the control vector will have the form:

$$\mathbf{u} = W_m + W_h - J_\phi \tag{17}$$

The required parameters for missile control, taking into account the control of the angle of the radar observation in the missile active radar homing head of the radar signals reflected from the helicopter and the angular velocity of rotation of the line of sight "missile-helicopter", are φ_T and ω_T, respectively, i.e. the vector of the required phase coordinates x_T will have the form:

$$x_T = \begin{bmatrix} \varphi_T \\ \omega_T \end{bmatrix} \tag{18}$$

The controlled parameters for the homing system of the missile are φ and ω, i.e. the vector of controlled phase coordinates x_y will be as follows:

$$x_Y = \begin{bmatrix} \varphi \\ \omega \end{bmatrix} \tag{19}$$

Since the control parameter is the lateral acceleration of the rocket j_p, and the controlled parameters are φ and ω, then the matrix of the efficiency of control signals By is formed by the coefficients of the control vector **u** in the dynamic model (16):

$$\mathbf{B}_Y^T = \begin{bmatrix} -\dfrac{1}{D} & C \end{bmatrix} \tag{20}$$

The matrices of penalties for the guidance accuracy **Q** and the magnitude of the control signals K in the formula (4) taking into account the dimensionality of the matrices (19)–(21) have the form

$$\mathbf{Q} = \begin{bmatrix} q_\varphi & 0 \\ 0 & q_\omega \end{bmatrix};\ \mathbf{K} = k \tag{21}$$

As a result, the quality functional (5) takes the form:

$$\mathbf{J} = M\left\{ \begin{bmatrix} \varphi_r - \varphi \\ -\omega \end{bmatrix}^T \begin{bmatrix} q_\varphi & 0 \\ 0 & q_\omega \end{bmatrix} \begin{bmatrix} \varphi_r - \varphi \\ -\omega \end{bmatrix} + \int_0^{t_i} \left(W_m + W_p - J_\phi\right)^2 k\,dt \right\} \tag{22}$$

Substituting (18)–(22) into (4), we obtain the following relationship:

$$W_m + W_p - J_\phi = \frac{1}{k} \begin{bmatrix} \dfrac{1}{\dot{D}} \\ \dfrac{1}{D} \end{bmatrix}^T \begin{bmatrix} q_\varphi & 0 \\ 0 & q_\omega \end{bmatrix} \begin{bmatrix} \varphi_T - \varphi \\ -\omega \end{bmatrix} = \frac{q_\varphi}{k\dot{D}}\left(\varphi_T - \varphi\right) - \frac{q_\omega}{kD}\omega \tag{23}$$

Or:

$$W_m = \frac{1}{k} \begin{bmatrix} \dfrac{1}{\dot{D}} \\ \dfrac{1}{D} \end{bmatrix}^T \begin{bmatrix} q_\varphi & 0 \\ 0 & q_\omega \end{bmatrix} \begin{bmatrix} \varphi_T - \varphi \\ -\omega \end{bmatrix} = \frac{q_\varphi}{k\dot{D}}\left(\varphi_T - \varphi\right) - \frac{q_\omega}{kD}\omega - W_p + J_\phi \tag{24}$$

This is the control signal and guidance law (24) for the missile to intercept the target.

3. Simulation and Evaluation

To evaluate the effectiveness of the proposed **optimal guidance law**, a nonlinear engagement simulation is conducted for an air-to-air missile (AAM) intercepting a **slow, highly maneuverable target** (e.g., helicopter or UAV). The missile and target dynamics are modeled in a planar engagement with realistic kinematic and control constraints.

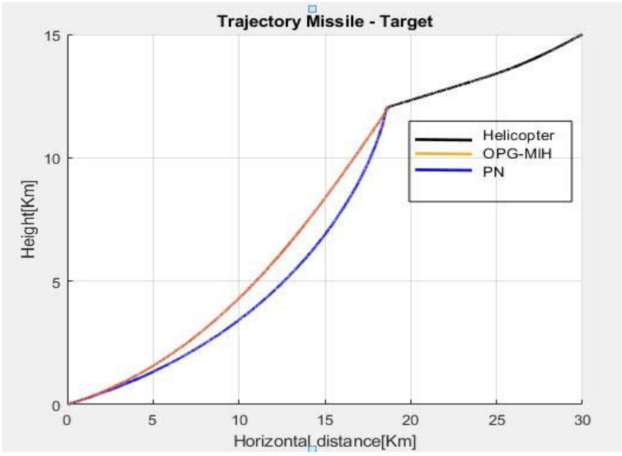

Figure 1.The trajectory of the missile and Target

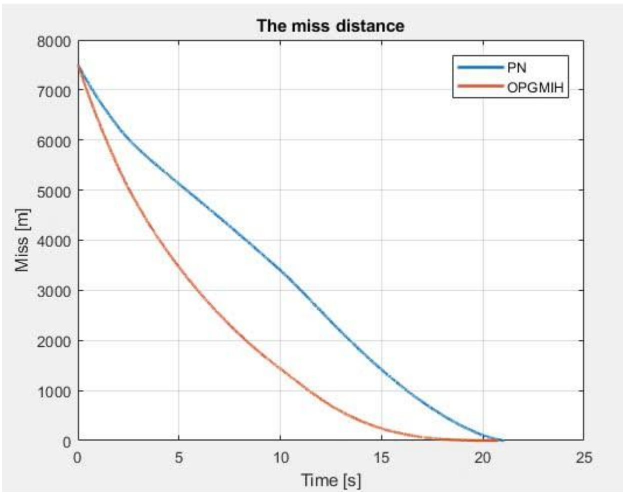

Figure 2.The miss distance

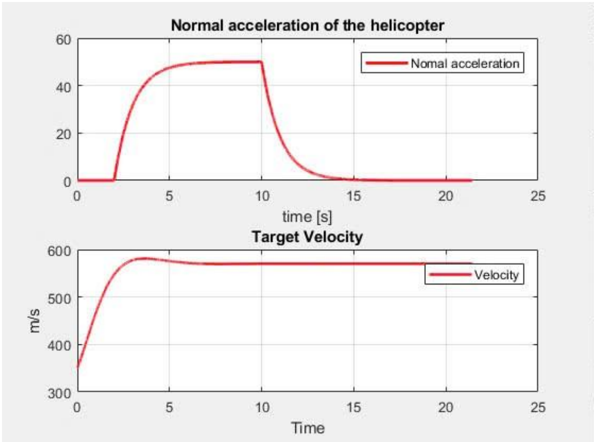

Figure 3. The change of velocity and normal acceleration of the helicopter

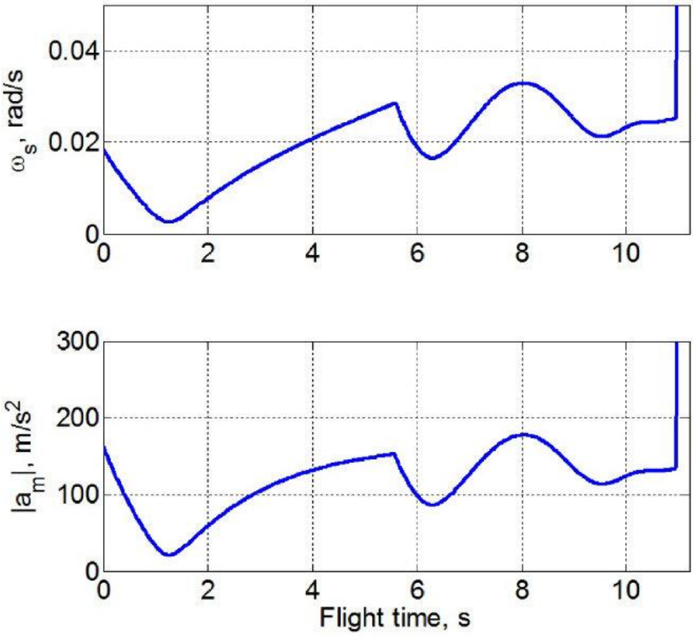

Figure 4. Missile Lateral Acceleration Command

The simulation results clearly demonstrate the effectiveness of the proposed optimal guidance law when engaging slow and highly maneuverable targets. Compared with classical proportional navigation (PN), the proposed method exhibits superior interception performance under aggressive target maneuvers.

4. Conclusion

This paper has presented an optimal guidance law for air-to-air missiles engaging slow and highly maneuverable targets, such as helicopters and low-speed unmanned aerial vehicles. Unlike classical proportional navigation, which is primarily effective against high-speed and weakly maneuvering targets, the proposed approach explicitly accounts for target maneuverability and reduced closing velocity.

By formulating the interception problem within an optimal control framework, a closed-loop guidance law with a proportional navigation–like structure was analytically derived. The resulting guidance law achieves an improved balance between interception accuracy and control effort, making it well suited for practical implementation in air-to-air missile systems.

Comprehensive simulation results demonstrate that the proposed guidance law significantly reduces terminal miss distance and missile lateral acceleration compared with classical proportional navigation under aggressive target maneuvers. The line-of-sight rate converges more rapidly, and the missile trajectory exhibits smoother behavior, confirming enhanced stability and robustness during the terminal phase of engagement.

These findings indicate that classical proportional navigation is not optimal for engagements involving slow and highly maneuverable targets. The proposed guidance law provides an effective and computationally efficient alternative for such challenging scenarios. Future work will focus on extending the proposed framework to three-dimensional engagements, incorporating actuator constraints and seeker noise, and integrating learning-based techniques for adaptive gain tuning in highly uncertain environments.

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Declaration of Conflicting Interests

The authors declare that there are no conflicts of interest regarding the research, authorship, or publication of this article.

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