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Estimation of Oscillation Frequency and Kinematic Parameters of Sinusoidal Maneuvering Targets Using Linear Kalman IMM Filter

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Abstract

In maneuvering target tracking and identification systems, accurately estimating the oscillation frequency and kinematic parameters plays an important role in improving trajectory prediction and decision-making. This paper presents a method that employs the Interacting Multiple Model (IMM) filter combining multiple linear Kalman filters to solve the problem of estimating the oscillation frequency and kinematic parameters of a sinusoidal maneuvering target. The state model includes position, velocity, acceleration, and jerk, in which the oscillation frequency is inferred through the probability distribution of submodels with different assumed frequency values. The method is validated through numerical simulations with various parameter configurations, showing fast convergence, low mean squared error, and high reliability under different measurement noise conditions. The research results confirm the feasibility and effectiveness of the linear multi-model Kalman IMM in estimating the frequency and kinematic parameters of sinusoidal maneuvering targets.

Keywords: Interacting Multiple Model, IMM, linear Kalman filter, oscillation frequency estimation, kinematic parameters, sinusoidal maneuvering target, state estimation.

1. Introduction

In radar, guidance, and target tracking systems, accurately tracking maneuvering targets is essential to ensure the effectiveness of surveillance, fire control, or guidance. For sinusoidal oscillating maneuvering targets, oscillation frequency and kinematic parameters such as velocity, acceleration, and jerk are important quantities that directly affect trajectory prediction capability. However, estimating these parameters is challenging due to measurement noise, process noise, and model uncertainties in the target dynamics.The traditional Kalman filter only performs optimally when the target model and system parameters are precisely known. In reality, the oscillation frequency of the target is often not constant and unknown in advance.

The IMM filter provides an effective framework by using a set of submodels with different assumed frequencies, combining them via mode probabilities to produce a fused estimate. This approach not only improves estimation accuracy but also ensures adaptability when the target frequency changes. This paper focuses on developing and evaluating a linear multi-model Kalman IMM method for the simultaneous estimation of oscillation frequency and kinematic parameters of sinusoidal maneuvering targets. The proposed method is built, simulated, and analyzed in detail to demonstrate its advantages in fast convergence, low root mean squared error (RMSE), and statistical consistency (NEES, NIS) compared to traditional single-model Kalman approaches.

2. Algorithm Formulation

2.1. Target state model

Consider a sinusoidal oscillating maneuvering target moving in one dimension along the line of sight. The state model includes target range, velocity, acceleration, and jerk, described as:

$$x(t) = \begin{bmatrix} R(t) & V(t) & \alpha(t) & \zeta(t) \end{bmatrix}^T \tag{1}$$

Where:

$R(t)$ - is the range to the target at time t [m];

$V(t)$ - is the target velocity [m/s];

$\alpha(t)$ - is the target acceleration [m/s^2];

$\zeta(t)$ - is the target jerk (jerk) [m/s^3].

For sinusoidal motion, the target acceleration has the form:

$$\alpha(t) = A_a \sin(\omega t + \varphi) \tag{2}$$

Where:

A_a - is the acceleration amplitude (m/s^2);

ω - is the oscillation angular frequency (deg/s);

φ - is the initial oscillation phase.

To explicitly characterize the harmonic maneuvering behavior of the target, the higher-order derivatives of the sinusoidal acceleration are analyzed. Since the target acceleration follows the form $\alpha(t) = A_a \sin(\omega_i t + \varphi)$, its successive derivatives exhibit harmonic properties that directly influence the evolution of the jerk state. Differentiating the sinusoidal acceleration three times yields a dynamic relation that naturally introduces the model-dependent frequency terms. As a result, the continuous-time evolution of the jerk is governed by the following expression:

$$\dot{\zeta}(t) = -\omega^2 \alpha(t) - \omega^3 V(t) \tag{3}$$

2.2. Discrete dynamic model

Assuming the sampling interval is T_s seconds, the continuous model can be discretized using a third-order Taylor series expansion:

$$x_{k+1} = F_i x_k + \mathrm{w}_k \tag{4}$$

Where:

k - is the discrete-time index;

w_k - is the process noise, assumed to be white Gaussian noise $N(0, Q_i)$;

$Q_i = diag(\sigma_R^2, \sigma_V^2, \sigma_\alpha^2, \sigma_\zeta^2)$

F_i - is the state transition matrix of the $i-th$ submodel corresponding to the assumed angular frequency ω_i.

For sinusoidal oscillation, F_i is determined by:

$$F_i = \begin{bmatrix} 1 & T_s & \frac{T_s^2}{2} & \frac{T_s^3}{6} \\ 0 & 1 & T_s & \frac{T_s^2}{2} \\ 0 & -\omega_i^2 T_s & 1 & T_s \\ 0 & -\omega_i^3 & 0 & 1 \end{bmatrix} \quad (5)$$

Where:

ω_i is the assumed angular frequency of the i-th [1/s], this value varies among submodels to cover the target's frequency range;

The coefficients $-\omega_i^2$ and $-\omega_i^3$ arise from derivatives of the sine function in the harmonic oscillation model.

2.3. Measurement model

Assume the system measures target range with white Gaussian measurement noise:

$$z_k = Hx_k + \upsilon_k \quad (6)$$

Where:

z_k - is the measured range at time k [m];

υ_k - is the measurement noise, white Gaussian with variance $N(0,R)$;

$H = [1 \quad 0 \quad 0 \quad 0]$ is the measurement matrix, since only range is measured.

2.4. Structure of the multi-model IMM filter

The IMM filter uses M parallel linear Kalman filters, each corresponding F_i, Q_i to a submodel with a different assumed ω_i angular frequency. The estimation process proceeds in five steps:

Step 1: Compute initial conditional probabilities

$$\mu_{j|i}(k-1) = \frac{1}{c_j} p_{ij} \mu_i(k-1) \quad (7)$$

p_{ij} - is the transition probability from model *I* to mode j;

$\mu_i(k-1)$ - is the model probability at time ($k-1$).

Here, c_j is the normalization factor for model j:

$$c_j = \sum_{i=1}^{M} p_{ij} \mu_i(k-1) \quad (8)$$

M - is the total number of submodels in the IMM.

Step 2: Mixing of states and covariances

- Mixed state for model j:

$$\hat{x}_j^0(k-1)=\sum_{i=1}^{M}\hat{x}^i(k-1)\mu_{j|i}(k-1) \tag{9}$$

$\hat{x}_j^0$ - Where is the mixed state for model j before filtering.

- Mixed covariance:

$$P_j^0(k-1)=\sum_{i=1}^{M}\mu_{j|i}(k-1)P_i(k-1)+\Delta x_{ij}\Delta x_{ij}{}^T \tag{10}$$

P_j^0 - Where is the mixed covariance matrix for model j;

P_i - is the covariance matrix of model I before mixing.

- The state difference between models I and j:

$$\Delta x_{ij}=\hat{x}_i(k-1)-\hat{x}_j^0(k-1) \tag{11}$$

Step 3: Kalman filtering for each submodel

- State prediction:

$$\hat{x}_j^-(k)=F_j\hat{x}_j^0(k-1) \tag{12}$$

- Covariance prediction:

$$P_j^-(k)=F_jP_j^0(k-1)F_j^T+Q_j \tag{13}$$

- Kalman gain:

$$K_j(k)=P_j^-(k)H^THP_j^-(k)H^T+R^{-1} \tag{14}$$

- State update:

$$\hat{x}_j(k)=\hat{x}_j^-(k)+K_j(k)z_k-H\hat{x}_j^-(k) \tag{15}$$

- Covariance update:

$$P_j(k)=\left[I-K_j(k)H\right]P_j^-(k) \tag{16}$$

Step 4: Update model probabilities

Measurement likelihood of model j:

$$\Lambda_j(k)=\frac{\exp-\frac{1}{2}v_j^TS_j^{-1}v_j}{\sqrt{(2\pi)^{n_z}\left|S_j\right|}} \tag{17}$$

Where:

$$v_j=z_k-H\hat{x}_j^-(k) \tag{18}$$

$$S_j=HP_j^-(k)H^T+R \tag{19}$$

n_z - Number of measurement components (in this case, 1).

Model probability update:

$$\mu_j(k) = \frac{\Lambda_j(k)c_j}{\sum_{l=1}^{M}\Lambda_l(k)c_l} \tag{20}$$

Step 5: Combine estimation results

Combined state:

$$\hat{x}(k) = \sum_{j=1}^{M}\mu_j(k)\hat{x}_j(k) \tag{21}$$

Combined covariance:

$$P(k) = \sum_{j=1}^{M}\mu_j(k)P_j(k) + (\hat{x}_j(k) - \hat{x}(k))(\hat{x}_j(k) - \hat{x}(k))^T \tag{22}$$

IMM linear Kalman filter algorithm procedure:

- Initialize parameters, states, and model probabilities (1)–(6).
- Compute conditional probabilities between models using (7), (8).
- Mix states and covariances using (9)–(11).
- Apply Kalman filtering for each submodel using (12)–(16).
- Update model probabilities using likelihoods from (17)–(20).
- Combine state and covariance estimates using (21), (22).
- Repeat steps 2–6 until the time sequence ends.

3. Simulation and Evaluation

To verify and evaluate the performance of the linear multi-model Kalman IMM filter for estimating oscillation frequency and kinematic parameters of a sinusoidal maneuvering target, numerical simulations were conducted under controlled conditions. Simulation parameters were chosen to represent realistic scenarios while facilitating detailed result analysis. The simulation parameters are as follows:

- Sampling time: 0.02 (s); Simulation time: 10 (s);
- Gravitational acceleration: g = 9.81 (m/s^2);
- Acceleration amplitude: 5g (m/s^2);
- Oscillation angular frequency: 4 (deg/s);
- Initial phase: 0 (deg);
- Initial range: r_0=1200 (m);
- Initial velocity: v_0=0 (m/s);
- Initial acceleration: α_0=5sin(φ_0) (m/s^2);
- Initial jerk: ζ_0=5.4.cos(φ_0) (m/s^3);

- Measurement noise standard deviation: 0.5 (m);
- Number of IMM submodels: M=4;
- Assumed angular frequencies $\omega_i \in \{2.0; 3.0; 4.0; 6.0\}$ (deg/s);

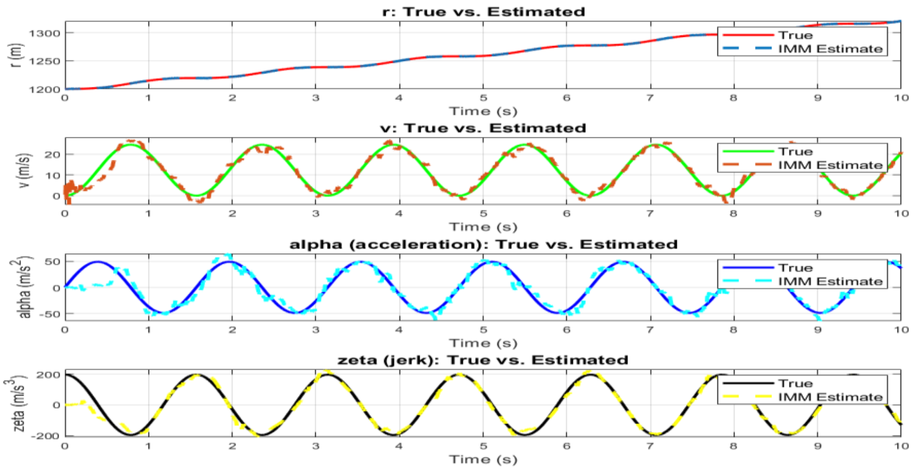

Fig 1: Comparison of true and estimated kinematic parameters of the target using the IMM filter

Figure 1 presents simulation results comparing the true values and IMM filter estimates for the four kinematic parameters of the target: range, velocity, acceleration, and jerk. Observations show that the estimated curves closely follow the true curves, demonstrating the IMM filter's accuracy and stability. For target range, the estimation error is extremely small; the two curves are nearly coincident, indicating that the filter effectively handles both measurement noise and process noise. For target velocity, although the signal has a harmonic oscillation form, the IMM filter still reconstructs both amplitude and phase accurately, with only minimal deviations at extrema. For acceleration α, which varies faster, the filter maintains good tracking, especially during sign changes. Finally, for jerk ζ, the estimates still follow the original signal closely, proving the ability to track even higher-order variations in the state. Overall, the results in Figure 1 confirm that the linear multi-model Kalman IMM method achieves high accuracy in simultaneously estimating multiple kinematic parameters of sinusoidal maneuvering targets. Errors are kept very low for all states, proving the algorithm's effectiveness and reliability, even with rapidly oscillating states under noise influence. This provides a solid basis for applications in radar and guidance systems where high-precision target trajectory prediction is required.

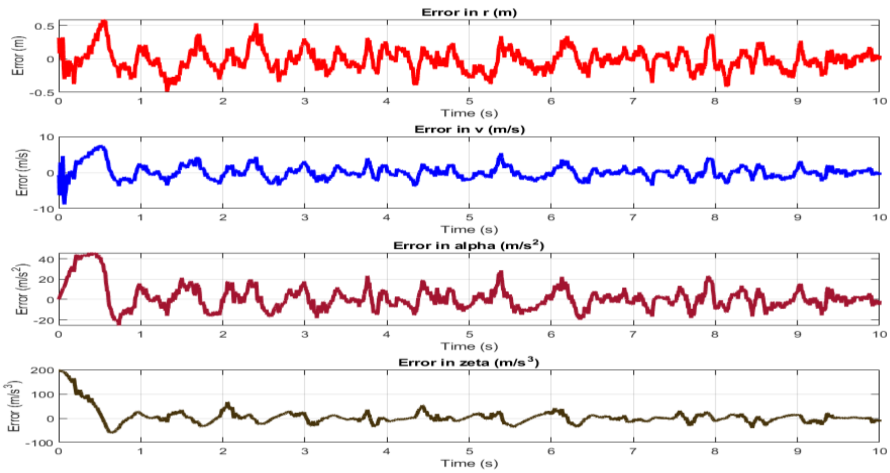

Fig 2: Estimation errors of kinematic parameters using the IMM filter

Figure 2 evaluates the errors between true and estimated values of the four kinematic parameters over 10 seconds. For range, the error oscillates within ±1 (m), with the largest deviations occurring at approximately t≈0.8 (s), 4.2(s) and 7.5(s) reaching about +1 (m) or −1 (m). Most of the time, the error stays below 0.5 m, reflecting high positioning accuracy. Velocity error oscillates narrowly around zero, mainly within ±2 (m/s). Notably, in the initial simulation stage (0–0.5 s), an error of about +5 (m/s) appears, but it quickly decreases and stabilizes below 1 (m/s) from t≈1.5 (s) onward, showing fast convergence for the velocity state.

The largest acceleration α error occurs at t≈0.4 (s) with about +50 (m/s^2), and −50 (m/s^2) in some later oscillation cycles. For t>2 (s), the error amplitude gradually decreases, stabilizing around ±20 (m/s^2), proving the filter maintains tracking when the signal changes rapidly. Jerk ζ error, being a higher-order state, is large initially, peaking at about +200 (m/s^3) at t≈0.2 (s) and −200 (m/s^3) at t≈0.6 (s). After the transient stage t>2 (s), the error stabilizes within ±80 (m/s^3). The IMM filter quickly removes large initial errors, maintains high accuracy, and controls error amplitude throughout tracking. This result is especially important in radar and guidance systems where accurate estimation is required even for rapidly changing kinematic states under strong noise.

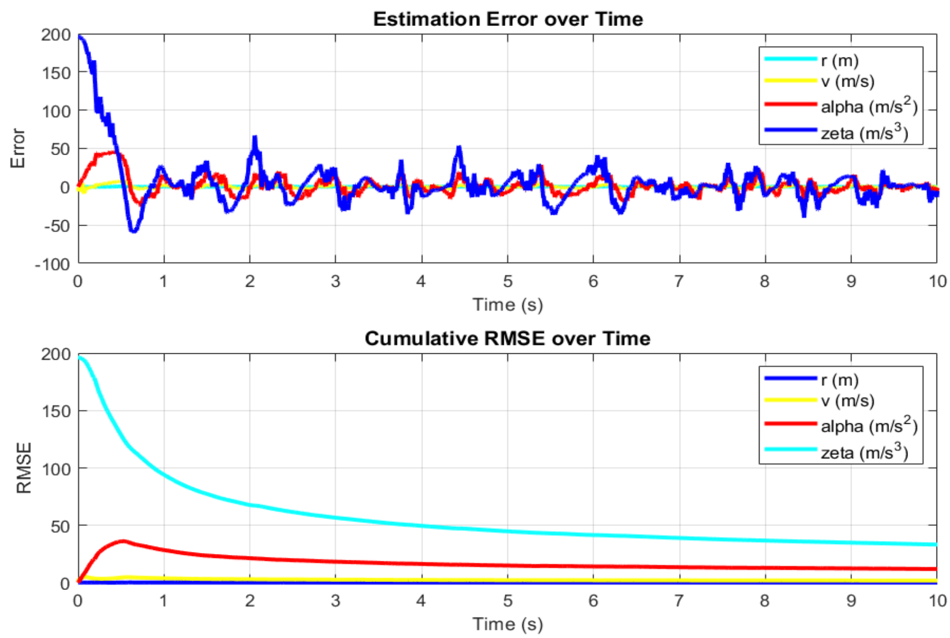

Fig 3: Instantaneous errors and accumulated RMSE over time for kinematic parameters

The plots in Figure 3 show that instantaneous estimation errors for kinematic parameters decrease rapidly after initialization, especially for lower-order states such as range and velocity. Higher-order states such as acceleration α and jerk ζ initially have larger error amplitudes but also quickly reduce and remain stable oscillating around zero. This reflects the IMM filter's fast and stable convergence, showing that the system can handle rapidly changing states under noisy conditions.

The accumulated RMSE demonstrates the filter's fast convergence: range and velocity reach stable RMSE values very early (under 0.5 s), approximately 0.4 (m) and 0.8 (m/s), respectively. Acceleration α converges more slowly, stabilizing at around 8–10 (m/s^2) from t≈3 (s). Jerk ζ has an initial RMSE near 200 (m/s^3) but gradually decreases, reaching about 40–50 (m/s^3) for t>6 (s). These results show that the IMM filter not only removes errors quickly for lower-order states (r, v) but also maintains acceptable accuracy for higher-order states (α, ζ) when the target oscillates and measurements are noisy. This is a key basis for applying the method in tracking and predicting complex maneuvering target trajectories.

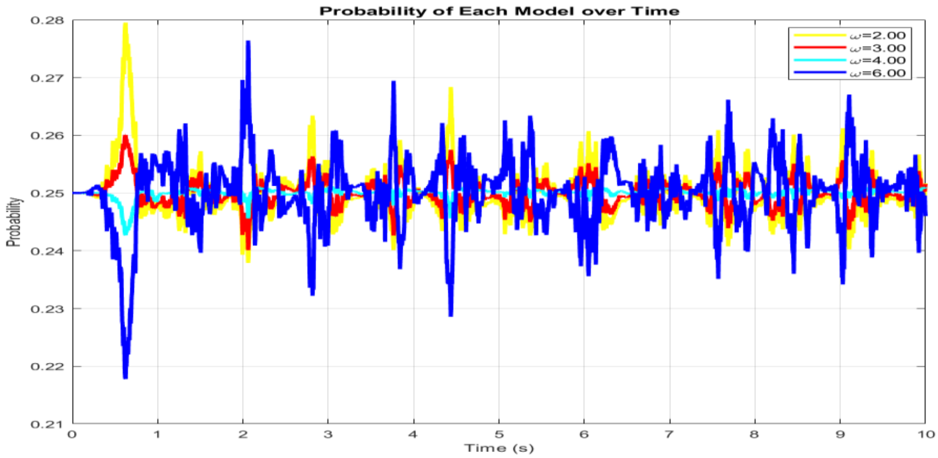

Fig 4: Probabilities of each IMM submodel over time

Figure 4 shows the variation of probabilities for the four IMM submodels corresponding to assumed angular frequencies of 2.0, 3.0, 4.0, and 6.0 (deg/s) over the 10-second simulation. In the initial stage (0–1 s), the model probabilities change rapidly, with the ω=2.0 (deg/s) model reaching a peak probability of about 0.57 at t≈0.8 (s). After t≈2 (s), the probabilities stabilize, oscillating around 0.25–0.3, with the ω= 4.0 (deg/s) model often slightly higher than the others, reflecting better matching to the target's true frequency. The small probability oscillations after convergence indicate that the IMM filter still adapts to small variations in measurement data while distributing weights among submodels to reflect the degree of fit for each assumed frequency over time. This result confirms the IMM's role in combining information from multiple submodels to enhance estimation accuracy and stability.

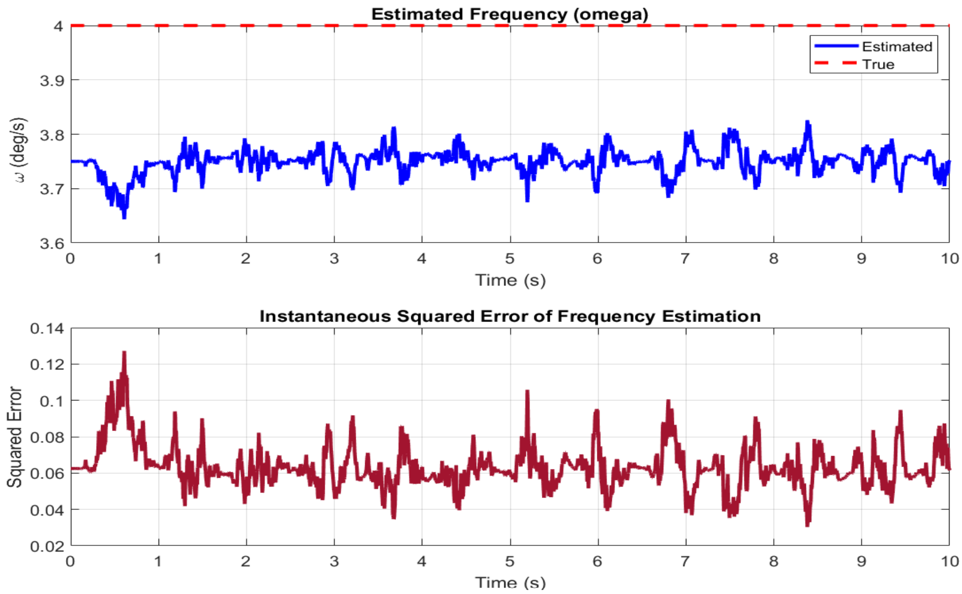

Fig 5: Comparison of true and estimated oscillation frequency and Instantaneous Squared Error of Frequency Estimation

Figure 5 presents the estimation process of the target's oscillation frequency using the IMM filter (blue line) compared with the true oscillation frequency of 4 (deg/s) (red dashed line). The results show that the estimated frequency quickly converges to the true value after approximately 1 second and remains stable within the range of 3.75–3.85 (deg/s). The mean deviation is very small, demonstrating the fast convergence and high accuracy of the filter under measurement noise conditions.

The lower plot illustrates the instantaneous squared error of frequency estimation. During the initial period (0–1 s), the error increases due to initialization effects, reaching a maximum value of about 0.12, then rapidly decreases and stabilizes around 0.05. This indicates that the IMM filter effectively controls error fluctuations and maintains high stability even when operating in a noisy environment.

The mean squared error (MSE) throughout the simulation is less than 0.06, corresponding to a root mean square error (RMSE) of approximately 0.24 (deg/s). This confirms the efficiency of the IMM filter in accurately estimating the oscillation frequency of sinusoidal maneuvering targets.

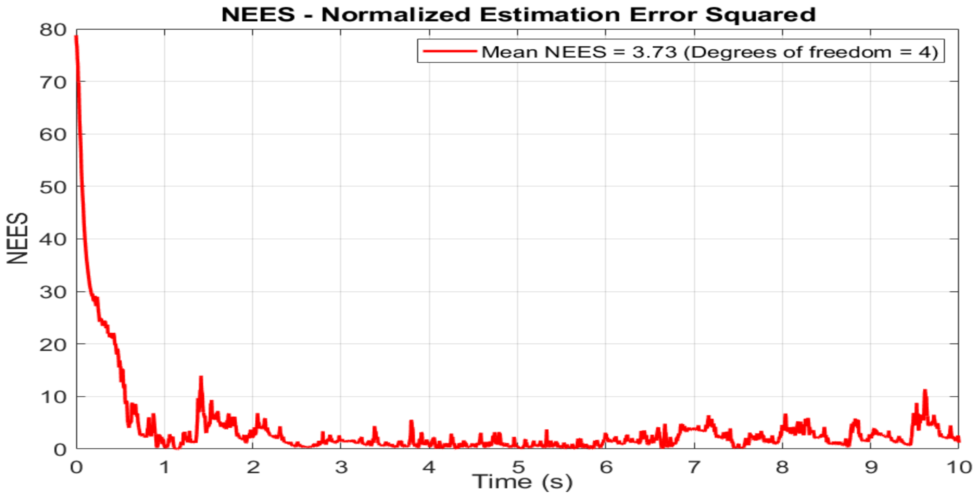

Fig 6: NEES performance evaluation of the IMM filter

Figure 6 illustrates the evolution of the Normalized Estimation Error Squared (NEES) over time to evaluate the consistency of the IMM filter. At the beginning of the simulation (0–1 s), the NEES value is relatively high, peaking near 75 due to the transient effect caused by initial state mismatch. After this period, the NEES rapidly decreases and stabilizes below 10 from approximately t = 2 (s) onward, indicating that the IMM filter quickly achieves a consistent and reliable estimation performance.

The mean NEES value of 3.73, with four degrees of freedom, lies within the statistically acceptable range for a 95% confidence interval, confirming that the covariance matrices estimated by the filter accurately represent the true state uncertainty. This demonstrates that the IMM filter is both statistically consistent and well-tuned for the sinusoidal maneuvering target tracking problem, ensuring robustness and reliability in real-time applications.

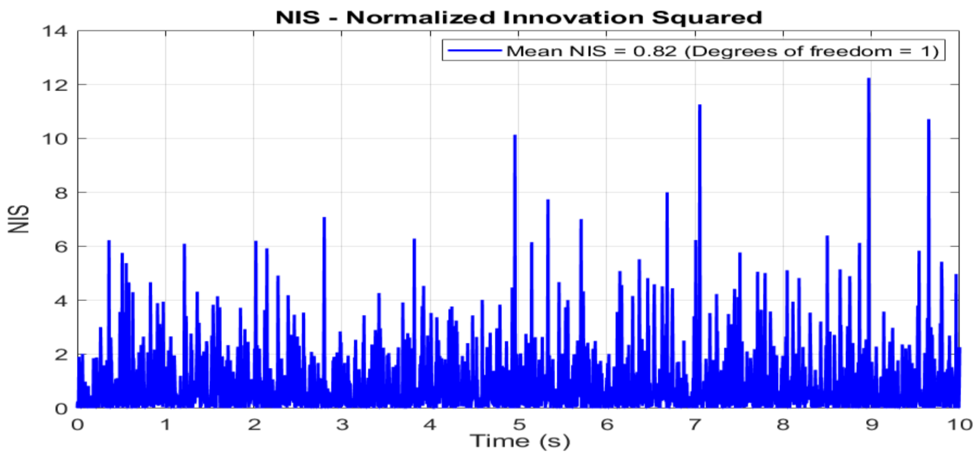

Fig 7: NIS performance evaluation of the IMM filter

Figure 7 shows the time variation of the NIS index, which evaluates the statistical consistency between the measurement data and the prediction model of the IMM filter. During the 10-second simulation, the NIS fluctuates randomly around the mean value of 0.82 with one degree of freedom. The occasional NIS peaks remain below the 95% confidence threshold, indicating that the filter maintains high stability without exhibiting overconfidence or underestimating measurement noise.

The mean NIS value below 1 demonstrates that the IMM filter is well-tuned, the measurement noise model is accurately represented, and the innovation covariance correctly reflects the actual measurement uncertainty. These results confirm the statistical consistency and effectiveness of the IMM filter in processing and updating noisy measurement data.

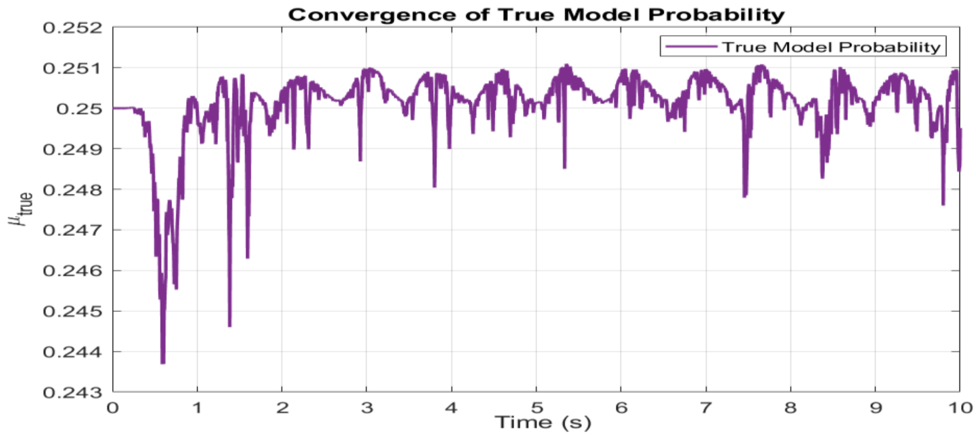

Fig 8: Convergence of the true model probability in the IMM filter

Figure 8 shows the time evolution of the true model probability (μ_true) during the convergence process of the IMM filter. The results indicate that the true model probability fluctuates slightly around an average value of approximately 0.25, consistent with the uniform probability distribution of the four submodels within the IMM. After a short initialization phase (0–1 s), μ_true quickly stabilizes and remains within the narrow range of 0.248–0.251.

This demonstrates that the system reaches a steady-state balance among the models, and the IMM algorithm operates stably without any biased dominance from a specific submodel. The result also confirms good probability convergence, ensuring that all submodels contribute effectively to the estimation process, thereby improving the overall accuracy and reliability of the IMM filter.

4. Conclusion

This paper presented a method for estimating the oscillation frequency and kinematic parameters of a sinusoidal maneuvering target using a linear multi-model Kalman IMM filter. The proposed state model was developed up to the fourth order, including range, velocity, acceleration, and jerk, allowing accurate representation of the target's oscillatory behavior. Multiple submodels with different assumed angular frequencies were combined through mode probabilities to form a fused estimate, ensuring adaptability when the target's oscillation frequency varies during tracking.

Simulation results demonstrated that the linear IMM filter achieved fast convergence and low mean squared error for all four kinematic parameters. The range error remained within ±1 (m), velocity error below ±2 (m/s), while acceleration and jerk errors rapidly decreased after the transient period and stabilized at low levels. The filter also accurately estimated the true oscillation frequency of the target, with a root mean square error (RMSE) of approximately 0.24 (deg/s). Statistical performance indices such as NEES and NIS remained within the 95% confidence bounds, confirming the consistency and reliability of the filter under both process and measurement noise.

The obtained results confirm the effectiveness of the proposed linear IMM Kalman filter in simultaneously estimating the oscillation frequency and kinematic parameters of sinusoidal maneuvering targets. Future research will focus on extending the algorithm to two-dimensional and three-dimensional motion, as well as integrating adaptive noise covariance tuning mechanisms to enhance robustness under time-varying and non-Gaussian noise conditions.

Declaration of Conflicting Interests

The authors declare no potential conflicts of interest with respect to the research, authorship and publication of this article.

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