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# A Synthesis of Stabilization Algorithms for Long-Range Surface-to-Air Missile Systems

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## Abstract

Long-range surface-to-air missile (SAM) systems operate under extreme aerodynamic, environmental, and electronic warfare conditions. Ensuring flight stability across the entire engagement envelope requires robust, adaptive, and computationally efficient algorithms. This paper presents a comprehensive synthesis of stabilization algorithms for long-range SAM systems, covering classical, modern, robust, and intelligent control methods. Mathematical models, stability analyses, and algorithmic formulations are included. A unified hierarchical control architecture combining linear, strong, and adaptive layers is proposed. Comparative evaluations and cited references support the theoretical discussion.

***Keywords:*** Long-range surface-to-air missile; synthesis of stabilization.

## 1. Introduction

The rapid evolution of modern military technology has led to the widespread deployment of advanced weapon systems, among which long-range surface-to-air missiles (SAMs) serve as the core strategic assets for national air-defense operations [1], [2]. A deep understanding of missile-control systems is therefore essential not only for mastering the operation of current platforms but also for enabling the design, development, and modernization of next-generation missile technologies [3]. Strengthening research in this field directly contributes to enhancing the tactical and technical performance of existing missile inventories and ensures readiness for emerging mission requirements.

In missile-control systems in general—and guided surface-to-air missiles in particular—the overall effectiveness of the engagement, measured by the probability of kill, strongly depends on the control error [4]. Reducing this error requires maintaining stable orientation of the missile throughout its flight, despite environmental disturbances and internal parameter variations [5]. The onboard stabilization system is thus an indispensable element of any missile or aerial vehicle. Its primary function is to preserve the missile's attitude, counteracting aerodynamic effects, structural flexibility, sensor noise, and propulsion-induced fluctuations [6]. As the fundamental component of the autopilot, the onboard stabilizer determines the missile's responsiveness, agility, and robustness during high-performance maneuvers [7].

For modern missiles characterized by high speeds and extreme aerodynamic conditions, static stability can be significantly reduced or even lost at certain flight regimes [8]. Under such conditions, control can only be guaranteed if an effective stabilizing system is integrated into the autopilot [9]. Traditional approaches to designing onboard stabilizers rely on linearizing the missile's nonlinear dynamics around specific operating points [10]. Linear or nonlinear control laws are then synthesized for each condition, and the resulting controller parameters are stored for use across the flight envelope. Although straightforward to implement, this method requires repeated linearization and gain scheduling, which may lead to performance degradation when the missile encounters highly nonlinear or rapidly changing conditions [11], [12].

The objective of this work is to investigate the fundamental aspects of missile onboard-stabilization problems and to apply modern control-theoretic tools—including analytical methods, normalized canonical forms, and PID-based synthesis—to develop stabilization controllers suitable for long-range SAM applications [13], [14]. The study aims to establish a structured approach for designing and implementing onboard stabilization systems that enhance missile agility, robustness, and precision throughout the engagement.

## 2. Missile Equations of Motion in Space

The motion of a rocket in space can be considered as the motion of a solid body with a translational center of gravity and rotational motion of the solid body around the center of gravity.

The translational motion of the center of gravity of a solid body moving in space is determined by the following theorem:

Theorem on momentum: For an object with a changing mass, the derivative of the momentum vector with time is equal to the sum of the external force vector F, the reaction force vector P, the Coliolis force Fk and the force due to the impact of the turbulent environment FB:

$$\frac{d\overline{Q}}{dt} = \overline{F} + \overline{P} + \overline{F}_k + \overline{F}_B \tag{1}$$

For a system with a constant mass, equation (1) can be written as:

$$\frac{d\overline{Q}}{dt} = \overline{F} \tag{2}$$

Postulate on angular momentum. For an object with a changing mass, the time derivative of the momentum vector about the center of gravity is equal to the sum of the moments of the forces acting on the rocket:

$$\frac{d\overline{K}}{dt} = \overline{M} + \overline{M}_K + \overline{M}_M \tag{3}$$

In which:

$\overline{K}$ - angular momentum

$\overline{M}$ - external moment

$\overline{M}_K$ - Coliolis moment

$\overline{M}_M$ - engine thrust moment

The vector derivative is equal to the linear velocity of that vector and the sum of its two components. The first component is the rate of change of the vector projection onto the axis of the moving coordinate system we are considering. The second component is equal to the product of the linear velocity due to its rotation along with the rotating coordinate system:

$$\frac{d\overline{V}}{dt} = \frac{d'\overline{V}}{dt} + \omega \times \overline{V} \tag{4}$$

In which:

$\frac{d'\nabla}{dt}$ - vector partial derivative

$\omega$ - angular velocity of the rotation of the moving coordinate system (velocity or link system) compared to the ground coordinate system.

## 3. Synthesis and Simulation Investigation of the Initial Launch Angle Stabilization System

Block Diagram of the Pitch Angle Automatic Stabilization System:

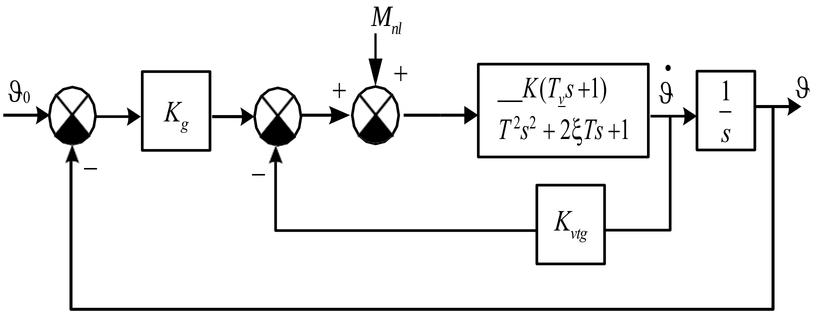


**Figure 1.** Structural diagram of the automatic nodding angle stabilization system

For the problem of stabilizing the initial launch angle, we synthesize and simulate to evaluate the quality of the stabilization system when the system input is the pre-calculated initial launch angle, and a disturbance torque acts on the missile.

The closed-loop transfer function of the stabilization system, considering the disturbance torque as the input to the missile (with $\vartheta_0 = 0$), is:

$$W_k(s) = \frac{\vartheta(s)}{M_{nl}(s)} = \frac{K(T_v s+1)}{T^2s^3 + (2\xi T + KT_vK_{vtg})s^2 + (KK_gT_v + KK_{vtg} + 1)s + KK_g} \tag{5}$$

And

$$\lim_{s\to 0} W_k(s) = \frac{\vartheta_{\max}}{M_{nl\max}} = \lim_{s\to 0} \frac{K(T_v s+1)}{T^2s^3 + (2\xi T + KT_vK_{vtg})s^2 + (KK_gT_v + KK_{vtg} + 1)s + KK_g} = \frac{1}{K_g} \tag{6}$$

This here:

$\vartheta_{\max}$ - is the error between the received nod angle and the initial launch angle. In terms of requirements, this error must be less than 0.5 antenna wave wings. We can take 0.5 antenna wave wings of the SAM missile as 2.50 = 0.0436rad. At this time, $\vartheta_{\max} < 0{,}0436 rad$ the missile is in the antenna wave wing area.

$M_{nl\max}$ - is the maximum disturbance moment acting on the rocket. Its value is calculated by the project designer. Suppose in this problem we can take it for convenience in the simulation and evaluation of the problem.

$$K_g > 229{,}4 \tag{7}$$

Thus, from equation (6), we can get the condition of the angle feedback coefficient:

Consider the input to the stabilization system as the initial launch angle according to the preset program. Let the disturbance moment acting on the rocket be 0.

The open transfer function of the nod angle stabilization system has the form:

$$W_h(\mathrm{p}) = \frac{K_gK(T_v s+1)}{T^2s^3 + (2T\xi + K_{vtg}KT_v)s^2 + (K_{vtg}K + 1)s} \tag{8}$$

Static error between the output nod angle and the input launch angle of the stabilization system:

$$e_{xl} = \lim_{s\to 0}\left(s.\frac{\vartheta_0(s)}{1+W_h(s)}\right) \tag{9}$$

From formula (8), we see that the open transfer function of the system has an integral term. To calculate the static error according to the angle, we let the input impact be a unit function. Static deviation at this time:

$$e_{xl} = \lim_{s\to 0}\left(s.\frac{\frac{1}{s}}{1+W_h(s)}\right) = \lim_{s\to 0}\left(\frac{1}{1+W_0(s).\frac{1}{s}}\right) = 0 \tag{10}$$

Thus, when there is no disturbance moment acting on the rocket, the static deviation between the exit angle and the initial launch angle is 0. In reality, there is always a disturbance acting on the rocket, causing the static deviation to be different from 0.

The closed-loop transfer function of the nod angle-stabilized system has the form:

$$W_k(\mathrm{p}) = \frac{K_g K(T_v s+1)}{T^2 s^3 + (2T\xi + K_{vtg} K T_v)s^2 + (K_{vtg} K + K_g K T_v + 1)s + K_g K} \tag{11}$$

To calculate the value of Kvtg, we need to know the limit range of Kvtg so that the closed-loop system is stable. At this time, we apply the Routh criterion to stabilize the system:
The characteristic equation of the closed-loop system has the form:

$$0{,}01s^3 + (0{,}01 + 0{,}05K_{vtg})s^2 + (K_{vtg} + 12{,}47)s + 229{,}4 = 0 \tag{12}$$

We proceed to create a Routh table:

**Table 1.** Routh Table

| | | | | |
|---|---|---|---|---|
| | $s^3$ | 0,01 | $K_{vtg}+12,47$ | 0 |
| | $s^2$ | $0{,}01+0{,}05K_{vtg}$ | 229,4 | 0 |
| $\alpha_3 = \frac{c_{11}}{c_{21}} = \frac{0{,}01}{0{,}01+0{,}05K_{vtg}}$ | $s^1$ | $K_{vtg}+12,47-\frac{0{,}01.229,4}{0{,}01+0{,}05K_{vtg}}$ | 0 | 0 |
| $\alpha_4 = \frac{c_{21}}{c_{31}}$ | $s^0$ | 229,4 | 0 | 0 |

The Routh criterion is stated as follows:

The necessary and sufficient condition for all solutions of the characteristic equation (10) to lie on the left side of the complex plane is that all elements in column 1 of the Routh table must be positive. The number of sign changes of the elements in column 1 of the Routh table is equal to the number of solutions lying on the right side of the complex plane.

Thus, applying the Routh criterion to stabilize the system, we have the following conditions:

$$\begin{gathered} 0{,}01+0{,}05K_{vtg} > 0; \\ K_{vtg}+12,47-\frac{0{,}01.229,4}{0{,}01+0{,}05K_{vtg}} > 0; \end{gathered} \tag{13}$$

Thus, Kvtg>2.8.

We choose Kg=229.4 and calculate the value of the feedback coefficient according to the angular velocity Kvtg.

To simplify the calculation of the Kvtg value, we choose the experimental method because this method is simple.

First, we choose Kvtg with any value Kvtg=15 and simulate to get the system quality index. From there, we get the value of Kvtg.

## 3. Simulation and Evaluation

To investigate the quality of the constructed algorithms, the paper simulates the results with the input parameters below.

**Table 2.** Angular velocity response coefficient table Kvtg

| No | $K_{vtg}$ | $T_{qd}$ | Number of oscillations | Overshoot |
|---|---|---|---|---|
| 1 | 5 | 0.6s | 5 | 37,6% |
| 2 | 10 | 0,24s | 2 | 3% |
| 3 | 15 | 0,3s | 0 | 0% |
| 4 | 20 | 0,4s | 0 | 0% |
| 5 | 25 | 0,47s | 0 | 0% |
| 6 | 30 | 0.52s | 0 | 0% |
| 7 | 35 | 0,65s | 0 | 0% |

From the parameters of Table 1, we see that choosing Kvtg=15 to get the best quality of the system, when reducing Kvtg, the oscillation increases, the overcorrection is large, when Kvtg increases, there is no oscillation; however, Tqd is large.

The simulation structure diagram on Simulink of the automatic launch angle stabilization system has the form:

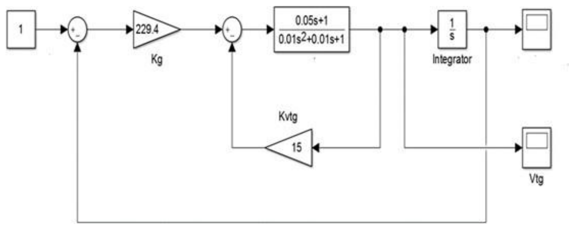


**Figure 2. Initial launch angle stabilization system when ignoring the effect of turbulent torque**

In Figure 2 shows the structure diagram of the initial launch angle stabilization system when there is no disturbance moment acting on the rocket.
The simulation results are as follows:

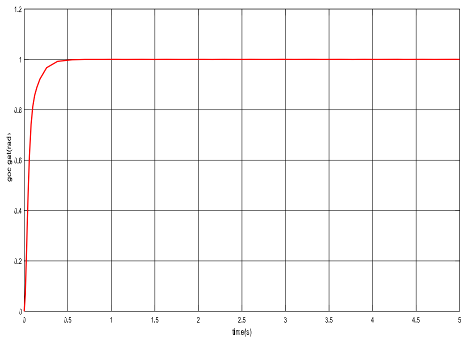


**Figure 3.** Nodding angle

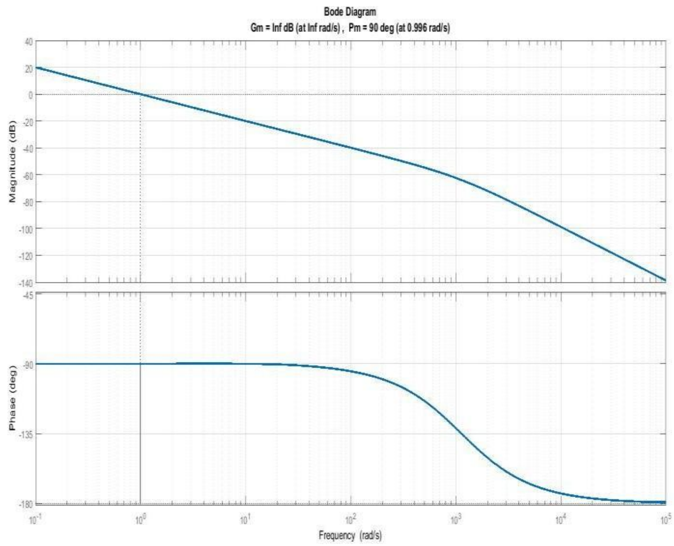


**Figure 4.** Open-loop logarithmic frequency characteristics of the auto-stabilization system initial launch angle

Simulation results:

+ Settlement time Txl=0.3s.
+ The system has no overcorrection.
+ Static error exl=0.
+ Phase margin Δφ= 900 at ωc=0.996 rad/s.

The system meets the required criteria well.

However, this is the result achieved when the system does not have a disturbance moment acting on the rocket. When there is a disturbance moment, the output harvest angle when established always deviates from the initial launch angle by a certain static deviation other than zero. The structure diagram of the system that automatically stabilizes the initial launch angle when there is a disturbance moment acting on the rocket input has the form:

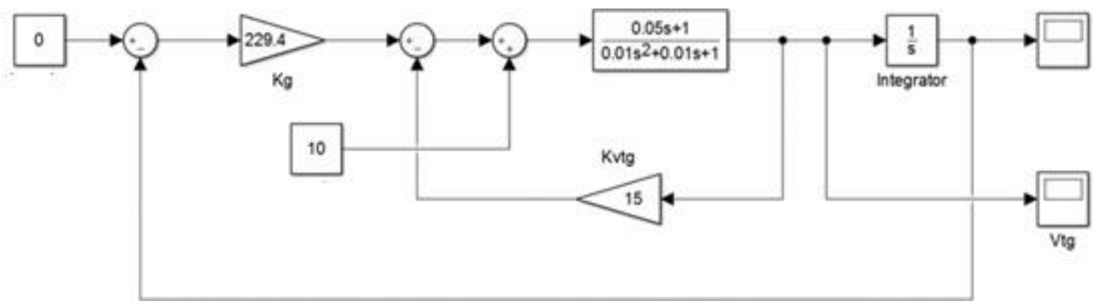


**Figure 5.** Schematic diagram of the initial launch angle structure when taking into account the influence of the perturbation moment on Simulink

The value Kg=229.4 corresponds to a static error limit between the output nod angle and the initial launch angle of 2.50=0.0436 rad.

Simulation results obtained:

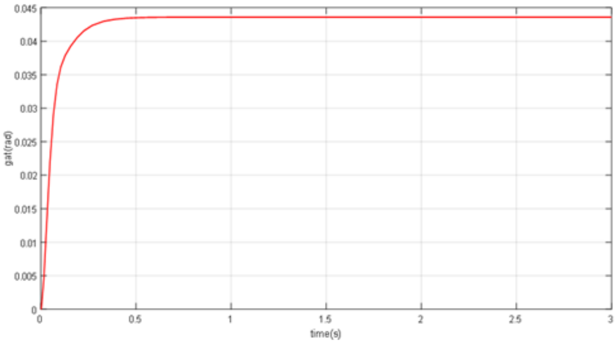


**Figure 6.** Stable system output in the presence of torque disturbance

From the simulation results, we see that the static deviation between the missile's yaw angle and the initial launch angle is exactly 0.0436 rad.

For the missile to be within the antenna wave, the static deviation must be less than 0.0436 rad. At this point, we must increase the value of Kg to reduce the static deviation.

Choose Kg=270 and proceed with the simulation to get the result:

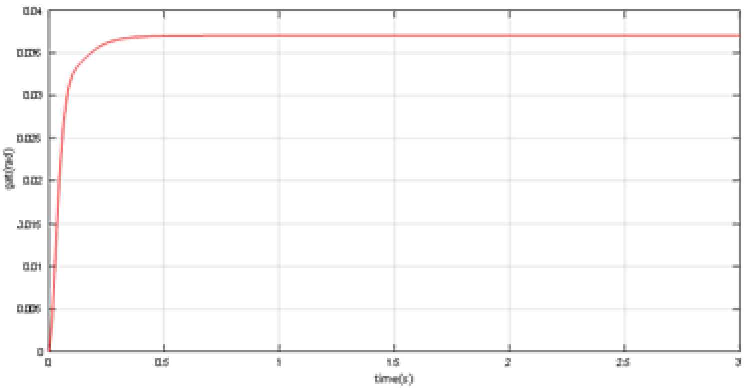


**Figure 7.** System output is stable when increasing Kg

Obviously, when increasing Kg, the static error value decreases to 0.037 rad, which is less than 2.50, the missile is now inside the antenna wave wing.

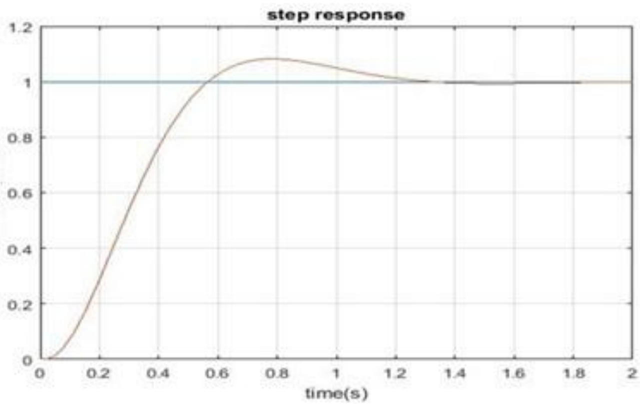


**Figure 8.** Pitch angle with overcorrection

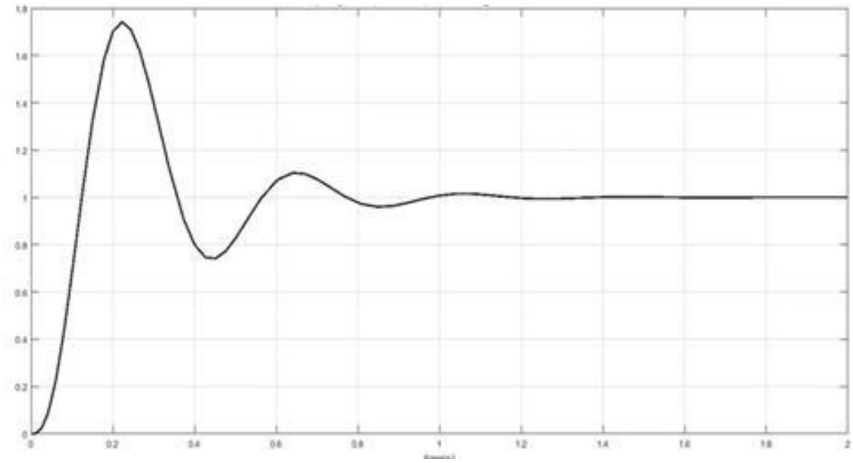

**Figure 9**. Transient response of the pitch angle ASS system using PID controller

Through the comparison table above, we can see that the synthesis of the automatic glide track stabilization system according to the normalized function model has much better quality than the PID method. From here, we can choose the normalized function model method to synthesize the design of the automatic glide track stabilization system.

## 4. Conclusion

This study investigated the influence of onboard stabilization system parameters on the dynamic characteristics of long-range missile guidance loops. For the internal roll-attitude stabilization problem, two controller-design approaches were examined: the normalized canonical-form method and the conventional PID-based method. Simulation results demonstrated that the normalized-function approach provides superior performance in roll-channel stabilization, offering faster response and improved robustness against parameter variations.

Another important focus of this work was the analysis of the output-limiting behavior of the rate gyroscope damper in the pitch-stabilization channel of the SAM missile. The results showed that the nonlinear saturation characteristics of the damper significantly affect the transient dynamics of the outer guidance loop. These effects were thoroughly evaluated through detailed simulations performed in MATLAB–Simulink for each stabilization subsystem.

Overall, the simulation findings confirm that the synthesized stabilization systems meet the required performance specifications, providing adequate responsiveness, stability margins, and disturbance-rejection capability. The results establish a solid foundation for further enhancement of onboard autopilot design and support future efforts in optimizing long-range surface-to-air missile guidance and control systems.

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## Declaration of Conflicting Interests

The authors declare that there are no conflicts of interest regarding the research, authorship, or publication of this article.

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