*International Journal of Science, Architecture, Technology, and Environment* *Volume 02, Issue 10, October 2025*
*ISSN 3048-8222 (Online) | www.ijsate.com | editor@ijsate.com*

International Journal of
Science, Architecture, Technology and Environment

# A Uniform Order Four-Step Hybrid Block Method for Numerical Solution of Susceptible-Infected-Recovered (SIR) and Growth Model

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*DOI: https://doi.org/10.63680/ijsate1025072.073*

## Abstract

In this research work, we derived a uniform order four step hybrid block method for the numerical solution of Susceptible-Infected-Recovered (SIR) and Growth model problems of ordinary differential equations (ODEs). A continuous linear multistep method (CLMM) with variable coefficients was developed using interpolation and collocation techniques via power series approximate solution as the basis function. This CLMM was evaluated at some selected grids points which give a class of discrete linear multistep methods (DLMMs) and was implemented as a block method. One case was considered for step numbers k=4. The basic properties of the block method were investigated and found out to be of order nine, consistent, zero stable and hence convergent. MATLAB 2015 codes were written to test the numerical performance of the block method on some real-life problems of ordinary differential equation and the results showed that the four-step hybrid block method compared favorably with the existing methods in terms of accuracy and efficiency, was found to be very effective.

***Keywords:*** Hybrid block method; continuous linear multistep method; numerical solution; ordinary differential equations; SIR model

## INTRODUCTION

The direct solution of first order initial value problem (IVP) of ordinary differential equations (ODEs) in form of (1.1) will be examined using linear multistep method in this study. First order ODEs are consider in solving real-life problems. Some natural phenomena are modeled using first order ODEs which are applied to most of the problems in applied sciences and engineering. Most of problems in the form of (1.1) it is challenging to apply analytic unless in a limited class. Consider the IVP in the form

$$y' = f(x, y(x_n)), y(x_0) = y_0 \qquad (1.1)$$

Most of the researchers give special consideration on first order initial value problem of ordinary differential equations. Among the authors are Awoyemi (1992), Adesanya, Alkali, Odekunle & Abubakar(2013), Yadav (2018) and Abolarin, Ogunware & Akinola (2020). Abolarin et al. (2020) developed an efficient seven-step block method for numerical solution of Susceptible Infected Recovered (SIR) and Growth Model using interpolation and collocation techniques for the solution of first order initial value problem of ordinary differential equations using power series as basis function. The existing method was carried out from to , interpolated at , collocated through the points and evaluated all the points except . Shown the block, continuous formulations, derivations of the block for finding discrete schemes and tested two known problems and results with others.

This research work is proposed to reduce the number of steps of Abolarin et al. (2020) to four-step and introducing four off-grid points to show the awareness of the students of Federal College of Education Gidan Madi about Susceptible class, to establish Infected class and Recovered class. The new numerical solution is expected to have higher accuracy and wider range of absolute stability error for better and more reliable results. The existing method iterations was carried out from zero to seven while thenew method iterations is from zero to four and introducing sub-intervals, so the number of iterations of thenew method is greater than the existing one. Definitely, the develop method guarantees better result than the existing method.

## 2.1 RESEARCH METHODOLOGY

## 2.2 METHOD OF DERIVATIVE

In improving the new method, power series in the form (1.2) will be considered as the trial solution to (1.1) where is the step size

$$y(x) = \sum_{j=0}^{m+t-1} c_j x^j \tag{1.2}$$

Equation (1.3) is obtained by differentiating (1.2)

$$y'(x) = \sum_{j=0}^{m+t-1} j c_j x^{j-1} \tag{1.3}$$

Interpolating (1.2) at $x_{n+j}, j = 0(4)$ and collocating (1.3) at $x_{n+j}, j = 0, \frac{1}{2}, \ldots, 4$ . The above equations are then combined to give a linear system of equations as obtained (1.4).

Using Gaussian elimination method for finding the values of $\alpha_{j's}$ , in (1.4) and substitute in (1.2) to obtain a continuous implicit scheme in the form

$$y(a) = \alpha_0(a) y_n + h\left(\sum_{j=0}^{m+t-1} \beta_j(a) f_{n+j}\right) \tag{1.5}$$

Where $y(a)$ is the trial solution of IVP, $\alpha_j$ and $\beta_j$ are both constant.

$$\sum_{j=0}^{m+t-1} \alpha_j y_{n+y} + \alpha_v y_{n+v} = h^k \left(\sum_{j=0}^{m+t-1} \beta_j f_{n+j} + \beta_v f_{n+v}\right) \tag{1.6}$$

$$\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 & x_n^9 \\ 0 & 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 \\ 0 & 1 & x_{n+\frac{1}{2}} & x_{n+\frac{1}{2}}^2 & x_{n+\frac{1}{2}}^3 & x_{n+\frac{1}{2}}^4 & x_{n+\frac{1}{2}}^5 & x_{n+\frac{1}{2}}^6 & x_{n+\frac{1}{2}}^7 & x_{n+\frac{1}{2}}^8 \\ 0 & 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 \\ 0 & 1 & x_{n+\frac{3}{2}} & x_{n+\frac{3}{2}}^2 & x_{n+\frac{3}{2}}^3 & x_{n+\frac{3}{2}}^4 & x_{n+\frac{3}{2}}^5 & x_{n+\frac{3}{2}}^6 & x_{n+\frac{3}{2}}^7 & x_{n+\frac{3}{2}}^8 \\ 0 & 1 & x_{n+2} & x_{n+2}^2 & x_{n+2}^3 & x_{n+2}^4 & x_{n+2}^5 & x_{n+2}^6 & x_{n+2}^7 & x_{n+2}^8 \\ 0 & 1 & x_{n+\frac{5}{2}} & x_{n+\frac{5}{2}}^2 & x_{n+\frac{5}{2}}^3 & x_{n+\frac{5}{2}}^4 & x_{n+\frac{5}{2}}^5 & x_{n+\frac{5}{2}}^6 & x_{n+\frac{5}{2}}^7 & x_{n+\frac{5}{2}}^8 \\ 0 & 1 & x_{n+3} & x_{n+3}^2 & x_{n+3}^3 & x_{n+3}^4 & x_{n+3}^5 & x_{n+3}^6 & x_{n+3}^7 & x_{n+3}^8 \\ 0 & 1 & x_{n+\frac{7}{2}} & x_{n+\frac{7}{2}}^2 & x_{n+\frac{7}{2}}^3 & x_{n+\frac{7}{2}}^4 & x_{n+\frac{7}{2}}^5 & x_{n+\frac{7}{2}}^6 & x_{n+\frac{7}{2}}^7 & x_{n+\frac{7}{2}}^8 \\ 0 & 1 & x_{n+4} & x_{n+4}^2 & x_{n+4}^3 & x_{n+4}^4 & x_{n+4}^5 & x_{n+4}^6 & x_{n+4}^7 & x_{n+4}^8 \end{bmatrix} \begin{bmatrix} a_0 \\ a_{\frac{1}{2}} \\ a_1 \\ a_{\frac{3}{2}} \\ a_2 \\ a_{\frac{5}{2}} \\ a_3 \\ a_{\frac{7}{2}} \\ a_4 \\ a_{\frac{9}{2}} \end{bmatrix} = \begin{bmatrix} y_n \\ f_n \\ f_{n+\frac{1}{2}} \\ f_{n+1} \\ f_{n+\frac{3}{2}} \\ f_{n+2} \\ f_{n+\frac{5}{2}} \\ f_{n+3} \\ f_{n+\frac{7}{2}} \\ f_{n+4} \end{bmatrix} \quad (1.4)$$

Using some algebraic manipulations to determine the values of as we obtained equation continuous formulation below

$$\alpha_0 = 1$$

$$\beta_0 = -\frac{1}{74723823216}\left(8521184t^9 - 326592t^8 - 1555417296t^7 + 15199702056t^6 - 67383418914t^5 + 165154346469t^4 - 233605716666t^3 + 185728156362t^2 - 74723823216t\right)$$

$$\beta_{\frac{1}{2}} = \frac{8}{16679424825}\left(1522360t^9 - 70875t^8 - 262533870t^7 + 2449131300t^6 - 10120465278t^5 + 22143967650t^4 - 25555876920t^3 + 12911950800t^2\right)$$

$$\beta_1 = -\frac{1}{44478466200}\left(85319360t^9 - 5140800t^8 - 13846242240t^7 + 123281087160t^6 - 476234354988t^5 + 941593371075t^4 - 924010635240t^3 + 349431077520t^2\right)$$

$$\beta_{\frac{3}{2}} = \frac{4}{16679424825}\left(10681720t^9 - 935550t^8 - 1622767770t^7 + 13795838910t^6 - 50059582146t^5 + 91035313425t^4 - 80520323520t^3 + 28106658720t^2\right)$$

$$\beta_2 = -\frac{1}{2668707972}\left(4292848t^9 - 725760t^8 - 604212984t^7 + 4915533168t^6 - 16873868277t^5 + 28825054146t^4 - 24158107953t^3 + 8123455494t^2\right)$$

$$\beta_{\frac{5}{2}} = \frac{4}{370653885}\left(1120t^9 - 19530t^8 + 140760t^7 - 542010t^6 + 1202544t^5 - 1540665t^4 + 1065960t^3 - 317520t^2\right)$$

$$\beta_3 = \frac{1}{133435398600}\left(84244160t^9 + 13608000t^8 - 10592536320t^7 + 78334540200t^6 - 246100772988t^5 + 390798120825t^4 - 310242712920t^3 + 100656486000t^2\right)$$

$$\beta_{\frac{7}{2}} = -\frac{8}{116755973775}\left(5290460t^9 + 411075t^8 - 605915865t^7 + 4327732710t^6 - 13222981233t^5 + 20567849400t^4 - 16092076140t^3 + 5171924520t^2\right)$$

$$\beta_4 = \frac{1}{88956932400}\left(6055840t^9 + 302400t^8 - 630753840t^7 + 4376528520t^6 - 13089549942t^5 + 20051121825t^4 - 15522477390t^3 + 4954939290t^2\right)$$

where $t = \dfrac{x - x_n}{h}$ evaluating the continuous hybrid scheme (3.5) at $x = x_{n+j}$, $j = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}$, and $4$ gives

### 2.3 Derivation of the Block

The combination of the normalize or discrete method are found by evaluating the continuous formulation at all the collocation points by using matrix inversion

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix} \begin{bmatrix} y_{n+\frac{1}{2}}\\ y_{n+1}\\ y_{n+\frac{3}{2}}\\ y_{n+2}\\ y_{n+\frac{5}{2}}\\ y_{n+3}\\ y_{n+\frac{7}{2}}\\ y_{n+4} \end{bmatrix} = \begin{bmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1 \end{bmatrix} \begin{bmatrix} y_{n-\frac{1}{2}}\\ y_{n-1}\\ y_{n-\frac{3}{2}}\\ y_{n-2}\\ y_{n-\frac{5}{2}}\\ y_{n-3}\\ y_{n-\frac{7}{2}}\\ y_{n} \end{bmatrix} + h \begin{bmatrix} 0&0&0&0&0&0&0&\frac{187021012373}{1195581171456}\\ 0&0&0&0&0&0&0&\frac{11177976613}{74723823216}\\ 0&0&0&0&0&0&0&\frac{2239620517}{14760261376}\\ 0&0&0&0&0&0&0&\frac{1401732463}{9340477902}\\ 0&0&0&0&0&0&0&\frac{186359484565}{1195581171456}\\ 0&0&0&0&0&0&0&\frac{150324413}{922516336}\\ 0&0&0&0&0&0&0&\frac{554034131}{3485659392}\\ 0&0&0&0&0&0&0&\frac{795160042}{4670238951} \end{bmatrix} \begin{bmatrix} f_{n-\frac{1}{2}}\\ f_{n-1}\\ f_{n-\frac{3}{2}}\\ f_{n-2}\\ f_{n-\frac{5}{2}}\\ f_{n-3}\\ f_{n-\frac{7}{2}}\\ f_{n} \end{bmatrix}$$

$$+h \begin{bmatrix} \frac{291188697701}{533741594400} & -\frac{282271069621}{711655459200} & \frac{16503194241}{533741594400} & -\frac{2859585769}{21349663776} & -\frac{1573169}{11860924320} & \frac{681515303631}{2134966377600} & -\frac{55460628751}{3736191160800} & \frac{3296645279}{1423310918400}\\ \frac{12541001336}{16679424825} & -\frac{294481847}{44478466200} & \frac{2979535156}{16679424825} & -\frac{115710341}{1334353986} & -\frac{37364}{370653885} & \frac{2950976957}{133435398600} & -\frac{1217879416}{116755973775} & \frac{146166703}{88956932400}\\ \frac{4805735733}{6589402400} & \frac{6567909921}{26357609600} & \frac{3181978073}{6589402400} & -\frac{34463277}{263576096} & -\frac{154953}{1317880480} & \frac{766227023}{26357609600} & -\frac{618367383}{46125816800} & \frac{109758861}{52715219200}\\ \frac{12406028032}{16679424825} & \frac{1041019402}{5559808275} & \frac{13485553472}{16679424825} & \frac{66738940}{667176993} & -\frac{36928}{370653885} & \frac{301502498}{16679424825} & -\frac{1063146752}{116755973775} & \frac{16453417}{11119616550}\\ \frac{14897654125}{21349663776} & \frac{9934552195}{28466218368} & \frac{10026525065}{21349663776} & -\frac{15930933175}{21349663776} & -\frac{427325}{2372184864} & \frac{9879211175}{85398655104} & -\frac{5868773255}{149447646432} & \frac{300598135}{56932436736}\\ \frac{132537528}{205918825} & \frac{870360147}{1647350600} & \frac{28480148}{205918825} & \frac{18111861}{16473506} & -\frac{111988}{41183765} & \frac{851578261}{1647350600} & -\frac{146267928}{144143731775} & \frac{39984597}{3294701200}\\ \frac{7331922773}{10892685600} & \frac{6338450027}{14523580800} & \frac{3174211033}{10892685600} & \frac{422322131}{435707424} & -\frac{52577}{242059680} & \frac{36910812463}{43570742400} & \frac{1310883791}{10892685600} & \frac{103459727}{29047161600}\\ \frac{9871345664}{16679424825} & \frac{3788617664}{5559808275} & -\frac{1601647616}{16679424825} & \frac{847902584}{667176993} & -\frac{167936}{370653885} & \frac{9771762496}{16679424825} & \frac{73881691136}{116755973775} & \frac{908664562}{5559808275} \end{bmatrix} \begin{bmatrix} f_{n+\frac{1}{2}}\\ f_{n+1}\\ f_{n+\frac{3}{2}}\\ f_{n+2}\\ f_{n+\frac{5}{2}}\\ f_{n+3}\\ f_{n+\frac{7}{2}}\\ f_{n+4} \end{bmatrix}$$

Showing all in a block forms of

$$y_{n+\frac{1}{2}} = y_n + h\left[\begin{array}{l} \frac{187021012373}{1195581171456} f_n + \frac{291188697701}{533741594400} f_{n+\frac{1}{2}} - \frac{282271069621}{711655459200} f_{n+1} \\ + \frac{165103194241}{533741594400} f_{n+\frac{3}{2}} - \frac{2859585769}{21349663776} f_{n+2} - \frac{1573169}{11860924320} f_{n+\frac{5}{2}} \\ + \frac{68151303631}{2134966377600} f_{n+3} - \frac{55460628751}{3736191160800} f_{n+\frac{7}{2}} + \frac{3296645279}{1423310918400} f_{n+4} \end{array}\right]$$

$$y_{n+1} = y_n + h\left[\begin{array}{l} \frac{11177976613}{74723823216} f_n + \frac{12541001336}{16679424825} f_{n+\frac{1}{2}} - \frac{294481847}{44478466200} f_{n+1} \\ + \frac{2979535156}{16679424825} f_{n+\frac{3}{2}} - \frac{115710341}{1334353986} f_{n+2} - \frac{37364}{370653885} f_{n+\frac{5}{2}} \\ + \frac{2950976957}{133435398600} f_{n+3} - \frac{1217879416}{116755973775} f_{n+\frac{7}{2}} + \frac{146166703}{88956932400} f_{n+4} \end{array}\right]$$

$$y_{n+\frac{3}{2}} = y_n + h\left[\begin{array}{l} \frac{2239620517}{14760261376} f_n + \frac{4805735733}{6589402400} f_{n+\frac{1}{2}} + \frac{6567909921}{26357609600} f_{n+1} \\ + \frac{3181978073}{6589402400} f_{n+\frac{3}{2}} - \frac{34463277}{263576096} f_{n+2} - \frac{154953}{1317880480} f_{n+\frac{5}{2}} \\ + \frac{766227023}{26357609600} f_{n+3} - \frac{618367383}{46125816800} f_{n+\frac{7}{2}} + \frac{109758861}{52715219200} f_{n+4} \end{array}\right]$$

$$y_{n+2} = y_n + h\left[\begin{array}{l} \frac{1401732463}{9340477902} f_n + \frac{12406028032}{16679424825} f_{n+\frac{1}{2}} + \frac{1041019402}{5559808275} f_{n+1} \\ + \frac{13485553472}{16679424825} f_{n+\frac{3}{2}} + \frac{66738940}{667176993} f_{n+2} - \frac{36928}{370653885} f_{n+\frac{5}{2}} \\ + \frac{301502498}{16679424825} f_{n+3} - \frac{1063146752}{116755973775} f_{n+\frac{7}{2}} + \frac{16453417}{11119616550} f_{n+4} \end{array}\right]$$

$$y_{n+\frac{5}{2}} = y_n + h\left[\begin{array}{l} \frac{186359484565}{1195581171456} f_n + \frac{14897654125}{21349663776} f_{n+\frac{1}{2}} + \frac{9934552195}{28466218368} f_{n+1} \\ + \frac{10026525065}{21349663776} f_{n+\frac{3}{2}} + \frac{15930933175}{21349663776} f_{n+2} - \frac{427325}{2372184864} f_{n+\frac{5}{2}} \\ + \frac{9879211175}{85398655104} f_{n+3} - \frac{5868773255}{149447646432} f_{n+\frac{7}{2}} + \frac{300598135}{56932436736} f_{n+4} \end{array}\right]$$

$$y_{n+3} = y_n + h\left[\begin{array}{l} \frac{150324413}{922516336} f_n + \frac{132537528}{205918825} f_{n+\frac{1}{2}} + \frac{870360147}{1647350600} f_{n+1} \\ + \frac{28480148}{205918825} f_{n+\frac{3}{2}} + \frac{18111861}{16473506} f_{n+2} - \frac{11988}{41183765} f_{n+\frac{5}{2}} \\ + \frac{851578261}{1647350600} f_{n+3} - \frac{146267928}{1441431775} f_{n+\frac{7}{2}} + \frac{39984597}{3294701200} f_{n+4} \end{array}\right]$$

$$y_{n+\frac{7}{2}} = y_n + h\left[\begin{array}{l} \frac{554034131}{3485659392} f_n + \frac{7331922773}{10892685600} f_{n+\frac{1}{2}} + \frac{6338450027}{14523580800} f_{n+1} \\ + \frac{3174211033}{10892685600} f_{n+\frac{3}{2}} + \frac{422322131}{435707424} f_{n+2} - \frac{52577}{242059680} f_{n+\frac{5}{2}} \\ + \frac{36910812463}{43570742400} f_{n+3} + \frac{1310883791}{10892685600} f_{n+\frac{7}{2}} + \frac{103459727}{29047161600} f_{n+4} \end{array}\right]$$

$$y_{n+4} = y_n + h\begin{bmatrix} \frac{795160042}{4670238951} f_n + \frac{9871345664}{16679424825} f_{n+\frac{1}{2}} + \frac{3788617664}{5559808275} f_{n+1} \\ -\frac{1601647616}{16679424825} f_{n+\frac{3}{2}} + \frac{847902584}{667176993} f_{n+2} - \frac{167936}{370653885} f_{n+\frac{5}{2}} \\ +\frac{9771762496}{16679424825} f_{n+3} + \frac{73881691136}{116755973775} f_{n+\frac{7}{2}} + \frac{908664562}{5559808275} f_{n+4} \end{bmatrix}$$

### 3.1 Analysis of the Basic Properties of the Block

### 3. 1. 1 Order and error constant

The block linear multistep method of first order ODEs is said to be of order p if $c_0 = c_1 = c_2 = \ldots = c_p = 0$ and $c_{p+1} \neq 0$ . Thus c's are the coefficient of h and y function, while $c_{p+1} \neq 0$ is called the error constant.

Now, the order of the method is of uniform order 9, with error constant

[-0.00002189133, -0.000016218011, -0.000019426768, -0.000015757438, -0.000033388274, -0.000058681122, -0.000039397666, -0.000010110999]

**Consistency of the method (Lambert, 1973)**

The block method Linear Multistep Method is said to be consistent if it has order $p = 9 > 1$

### 3. 1. 2 Zero stability of the method (Gebrigiorgis& Muleta, 2020)

The method is said to be zero stable if the roots z_s,s=1,2,3….,n of the first characteristics

polynomial defined by $\bar{p}(\lambda) = det\left[\lambda A^{(1)} - A^{(0)}\right] = 0$

Where

$$\lambda\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix} - \begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1 \end{pmatrix} = \begin{pmatrix} \lambda&0&0&0&0&0&0&-1\\ 0&\lambda&0&0&0&0&0&-1\\ 0&0&\lambda&0&0&0&0&-1\\ 0&0&0&\lambda&0&0&0&-1\\ 0&0&0&0&\lambda&0&0&-1\\ 0&0&0&0&0&\lambda&0&-1\\ 0&0&0&0&0&0&\lambda&-1\\ 0&0&0&0&0&0&0&\lambda-1 \end{pmatrix}$$, determinant: $\lambda^8 - \lambda^7 = \lambda^7\left(\lambda - 1\right)$

Therefore, λ_1=λ_2=λ_3=λ_4=λ_5=λ_6=0,λ_7=1

### 3. 1. 3 Convergence of the method (Lambert, 1973)

The Linear Multistep Method is said to be convergent if is zero stable and since it is consistent with order hence by Henrici (1962) the method is convergent.

**Testing the Efficiency of the Method**

**3. 1. 4 Tested problem 1 (SIR Model):**

The susceptible infected recovered disease is an epidemical disease which calculates the total of individuals that are sick with a transmissible infection in a populace over a period of time. such models develop from the fact that they contain coupled equations about the number of persons who are susceptible to disease is appropriately large within the host S(t), number of individual whose level of parasite I(t) and the whole individuals who have recovered R(t).

The variables are : $S(t)$ -Susceptible

$I(t)$ -Infected

$R(t)$ -Recovered

The model is coupled as:

$$\frac{dS}{dt} = \mu - \beta SI - \mu S \quad (4.16)$$

$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I \quad (4.17)$$

$$\frac{dR}{dt} = \gamma I - \mu R \quad (4.18)$$

for $\mu, \gamma$ *and* $\beta$ are positive parameters. $y$ is given as,

$$y = S + I + R \quad (4.19)$$

Summing (4.16), (4.17) and (4.18) to get

$$y' = \mu(1 - y) \quad (4.20)$$

let $\mu = 0.5$ with initial condition as $y(0) = 0.5$ , we get,

$$\frac{dy}{dt} = \frac{1}{2}(1 - y),\ y(0) = -\frac{1}{2},\ h = 0.1 \quad (4.21)$$

with exact solution:

$$y(t) = 1 - 0.5e^{-.0.5t} \quad (4.22)$$

### 3. 1. 5 Tested problem 2 (Growth Model):

Consider a growth model also known as bacteria culture is growing at a rate proportional to the whole present. One hour later, about 1000 strands of the bacteria are detected in the culture; and four hours later, 3000 strands of the bacteria also found. Calculate the total of strands of the bacteria available in the culture at time $t : 0 \leq t \leq 0.7$.

We assumed $y(t)$ to be the number of bacteria strands in the culture at time , the equation is modeled as

$$\frac{dy}{dt} = 0.366y,\ y(0) = 694 \qquad (4.23)$$

With particular solution as,

$$y(t) = 694e^{0.366t} \qquad (4.24)$$

Table 3.1: Comparison of the of test problem 1 with New Method, Abolarin et al.(2020) and Sunday et al. (2013)

| *X* | Exact Solution | Computed Results | New Method | Abolarin *et al.* (2020) | Sunday *et al.* (2013) |
|---|---|---|---|---|---|
| 0.1 | 0.5243852877496432 | 0.5243852877496430 | 2.220446e-16 | 9.103829e-015 | 5.574430e-012 |
| 0.2 | 0.5475812909820206 | 0.5475812909820202 | 3.330669e-16 | 7.105427e-015 | 3.946177e-012 |
| 0.3 | 0.5696460117874717 | 0.5696460117874711 | 5.551115e-16 | 8.881784e-015 | 8.183232e-012 |
| 0.4 | 0.5906346234610098 | 0.5906346234610092 | 6.661338e-16 | 2.120526e-014 | 3.436118e-011 |
| 0.5 | 0.6105996084642984 | 0.6105996084642975 | 8.881784e-16 | 1.367795e-013 | 1.929473e-010 |
| 0.6 | 0.6295908896591420 | 0.6295908896591411 | 8.881784e-16 | 7.982504e-013 | 1.879040e-010 |
| 0.7 | 0.6476559551406443 | 0.6476559551406433 | 9.992007e-16 | 3.698819e-012 | 1.776835e-010 |

**Tested problem 2 (Growth Model):**

Consider a growth model also known as bacteria culture is growing at a rate proportional to the whole present. One hour later, about 1000 strands of the bacteria are detected in the culture; and four hours later, 3000 strands of the bacteria also found. Calculate the total of strands of the bacteria available in the culture at time $t : 0 \leq t \leq 0.7$ .

We assumed $y(t)$ to be the number of bacteria strands in the culture at time , the equation is modeled as

| $x$ | Exact Solution | Computed Results | New Method | Abolarin *et al.* (2020) |
|---|---|---|---|---|
| 0.1 | 719.8709504841319 | 719.8709504841320 | 1.136868e-13 | 6.821210e-013 |
| 0.2 | 746.7063189494631 | 746.7063189494633 | 1.136868e-13 | 6.821210e-013 |
| 0.3 | 774.5420569951835 | 774.5420569951837 | 1.136868e-13 | 6.821210e-013 |
| 0.4 | 803.4154564251548 | 803.4154564251551 | 2.273737e-13 | 3.410605e-012 |
| 0.5 | 833.3651992080963 | 833.3651992080966 | 2.273737e-13 | 7.503331e-012 |
| 0.6 | 864.4314093001876 | 864.4314093001879 | 2.273737e-13 | 5.024958e-011 |
| 0.7 | 896.6557063995155 | 896.6557063995159 | 4.547474e-13 | 2.373781e-010 |

$$\frac{dy}{dt} = 0.366y,\ y(0) = 694 \qquad (4.23)$$

With particular solution as,

$$y(t) = 694e^{0.366t} \qquad (4.24)$$

Table 3.2 : Presentation of the methods when solving problem 2 with that of New Method andAbolarin et al.(2020)

### 3. 1. 6 Discussion

Tested problem 1 was solved by Abolarin et al.(2020), Sundayet al. (2013), called a SIR Model is an epidemical disease which calculates the total of individuals that are infected with a transmissible infection in a closed population over a period of time. From table 4.2 our method compute favorable than that of Abolarin et al. (2020), Sunday et al. (2013) when solving similar problem. Problem 2 is a Growth Model is also called a bacteria culture is growing at a rate proportional to the whole present, was solved by new method & Abolarin et al. (2020) and table 4.3 obviously showed the convergence of our method.

## Declaration of Conflicting Interests

The authors declare no potential conflicts of interest with respect to the research, authorship and publication of this article.

## Funding

The author received no financial support for the research, authorship and publication of this article.

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