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Select an Equation to Calculate of Average Energy of Bremsstrahlung Spectrum According to Al Filter Thickness for Rigaku-200EGM X-Ray Generator

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Abstract

Unlike gamma rays emitted from radioactive isotopes, which have fixed energies, bremsstrahlung X-rays generated by X-ray machines have variable average energies that depend on the type and thickness of the filter used. This paper presents a method for deriving an equation that describes the relationship between the average energy of the bremsstrahlung spectrum, the Al filter thickness, and the high voltage applied to the Rigaku-200EGM X-ray generator, which is installed at the Training Center of the Dalat Nuclear Research Institute in Vietnam. The results indicate that the average energy of the bremsstrahlung X-ray spectrum produced by the Rigaku-200EGM generator is a quadratic function of both the Al filter thickness and the high voltage supplied to the X-ray tube.

Keywords: Rigaku-200EGM; Bremsstrahlung; X-ray spectrum; Average energy; Al filter thickness

1. Introduction

The average energy of bremsstrahlung radiation produced by an X-ray generator is a crucial factor in characterizing the quality and biological impact of the emitted X-ray beam. Bremsstrahlung, or "braking radiation," occurs when high-speed electrons decelerate upon interacting with the target material, producing a continuous spectrum of X-ray photons ranging from near zero to the maximum kinetic energy of the electrons, which is determined by the tube voltage [1, 2]. The average energy effectively summarizes this spectrum as a representative photon energy value that strongly influences dosimetric calculations, radiation shielding requirements, and image quality. A higher average energy corresponds to a harder, more penetrating beam, which requires more substantial shielding but also results in better contrast and a reduced patient dose when appropriately filtered [1-3]. Moreover, the average energy determines the mass energy-absorption coefficients used to convert photon fluence into absorbed dose, a fundamental step in ensuring safe operation and compliance with radiation protection standards [4, 5]. By removing low-energy photons, filters such as aluminum (Al) and beryllium raise the average energy, thus optimizing the beam's clinical and industrial utility while minimizing unnecessary exposure [3]. Therefore, understanding and accurately determining the average energy of bremsstrahlung radiation is essential for effective X-ray system design, safety evaluation, and application.

To support teaching, the Rigaku-200EGM X-ray generator was installed at the Training Center of the Dalat Nuclear Research Institute in Vietnam. Unlike gamma rays from radioactive isotopes, which have fixed energies, bremsstrahlung X-rays produced by X-ray generators have an average energy that varies according to the type and thickness of the filter used. This research aims to identify an equation for calculating the bremsstrahlung average energy according to the Al filter thickness and the high voltage for the Rigaku-200EGM X-ray generator.

2. Materials and Methods

2.1. Materials

The Rigaku-200EGM X-ray generator, which has been described in detail in our previous investigations [2, 5], is an industrial X-ray system designed for nondestructive testing, operating in the voltage range of 70 kV to 200 kV with 2 kV steps. It features a ceramic X-ray tube with a tungsten anode and a 40-degree irradiation angle to produce a focused bremsstrahlung X-ray spectrum. The generator incorporates inherent filters, including a 1 mm thick beryllium window and a 2 mm thick Al filter, which help harden the beam by removing low-energy photons and improving penetration.

2.2. Methods

In our previous work [2], we showed that the Kramer model is the most accurate representation of the bremsstrahlung X-ray spectrum from the Rigaku-200EGM X-ray generator. This model closely matches experimental dose rate data provided by Rigaku Corporation, confirming its validity. It provides a reliable theoretical basis for spectral shape that aligns well with practical observations. This makes the Kramer model highly suitable for calculating key parameters such as dose rates and spectrum hardness in the Rigaku-200EGM system. The spectral distribution of the number dN of bremsstrahlung X-rays with energies ranging from E to E + dE, which corresponds to the Kramer model:

$$dN = k\left(\frac{E_0}{E} - 1\right)dE \qquad (1)$$

where k is the constant; E_0, E are the maximum energy and reference energy of the X-ray spectrum, respectively; and x is the correction value [2].

In this investigation [2], we used the integral ratio method, which was validated in our previous research, to calculate the average energy of the bremsstrahlung spectrum emitted by the Rigaku-200EGM X-ray generator, as expressed in the following equation:

$$\overline{E}_X = \frac{\int_0^{E_{\max}} E\Phi(E)dE}{\int_0^{E_{\max}} \Phi(E)dE} \qquad (2)$$

where, $\overline{E}_X$ = average energy of the bremsstrahlung X-ray spectrum (keV), E = energy of the bremsstrahlung X-ray (keV), and $\Phi(E)$ = function representing the bremsstrahlung X-ray spectrum.

3. Results and Discussion

3.1. Average energy of bremsstrahlung X-ray spectrum

Table 1 presents the calculated average energy of the bremsstrahlung X-ray spectrum. These calculations were conducted for different thicknesses of Al filters applied to the Rigaku-200EGM generator, using the spectral distribution described by the Kramer model.

Table 1. Calculated average energy of bremsstrahlung X-ray spectra according to Al filter thickness

T_e [kV]	Al filter thickness [mm]					
	0	1	2	4	6	8
70	23.0	31.9	35.7	40.2	42.3	43.9
80	25.2	34.4	39.0	43.6	45.8	47.8
100	29.5	39.2	44.0	49.4	52.5	54.4
150	37.6	50.0	55.2	61.3	64.5	67.1
200	44.9	59.4	66.4	72.1	75.7	79.7

The data in Table 1 demonstrate the calculated average energy of bremsstrahlung X-ray spectra at various tube voltages (T_e) ranging from 70 kV to 200 kV, with differing Al filter thicknesses varying from 0 to 8 mm. A clear trend emerges showing that the average energy of the X-ray spectra increases with increasing filter thickness for each tube voltage. At a fixed tube voltage, the average energy of the X-ray beam increases progressively as the Al filter thickness increases. For example, at 100 kV, the average energy increases from 29.5 keV without any filter to 54.4 keV with an 8 mm Al filter. This increase results from the preferential attenuation of lower-energy photons by the Al filter, a well-known phenomenon in X-ray beam hardening [1]. The filter removes low-energy bremsstrahlung photons more effectively, thereby increasing the mean energy of the transmitted spectrum.

Similarly, as the tube's high voltage increases, the average energy without filtration also rises. For example, at 70 kV without a filter, the average energy is 23.0 keV, compared to 44.9 keV at 200 kV. This is consistent with the physical phenomenon of bremsstrahlung production, where higher accelerating voltages produce higher energy X-rays [6]. The combined effect of high voltage and filter thickness means that for clinical or industrial applications requiring higher average photon energy, selecting higher voltage and appropriate filtration is crucial. For example, at the maximum tube voltage of 200 kV with an 8 mm Al filter, the average energy reaches 79.7 keV, which would notably enhance beam penetration for thick or dense materials [7]. These findings align with established literature on adjusting X-ray beam quality through filtration [8].

3.2. Select an equation for calculating average energy

To select an equation that facilitates the calculation of the mean energy of the bremsstrahlung spectrum, Origin software was used to fit the data presented in Table 1. The nonlinear poly2D function was employed to develop a mathematical model using surface fitting. This model, formulated in the study, estimates the average energy of the bremsstrahlung X-ray spectrum based on two variables of the Al filter thickness and the high voltage, as presented in Table 1 and Eq. (3). The result of the fitted model is described as:

$$\bar{E}_X = 8.10477 + 3.6701t + 0.31228T_e - 0.27339t^2 - 3.77132\times10^{-4}T_e^2 + 0.00746tT_e \qquad (3)$$

where t and T_e are the Al filter thickness (mm) and high voltage (kV), respectively.

To evaluate the reliability of the fitted model, the correlation coefficient R^2 indicates how well the regression model fits the data, while the percentage difference between the model's results and the calculated data is presented in Figure 1 and Table 2, respectively. The fitted model showed a strong fit with an R^2 value of 0.997, demonstrating its effectiveness in accurately predicting the mean energy based on the given parameters. This process produces a clear and precise mathematical model for estimating the mean energy of the bremsstrahlung X-ray spectrum across different Al filter thicknesses and high voltage levels.

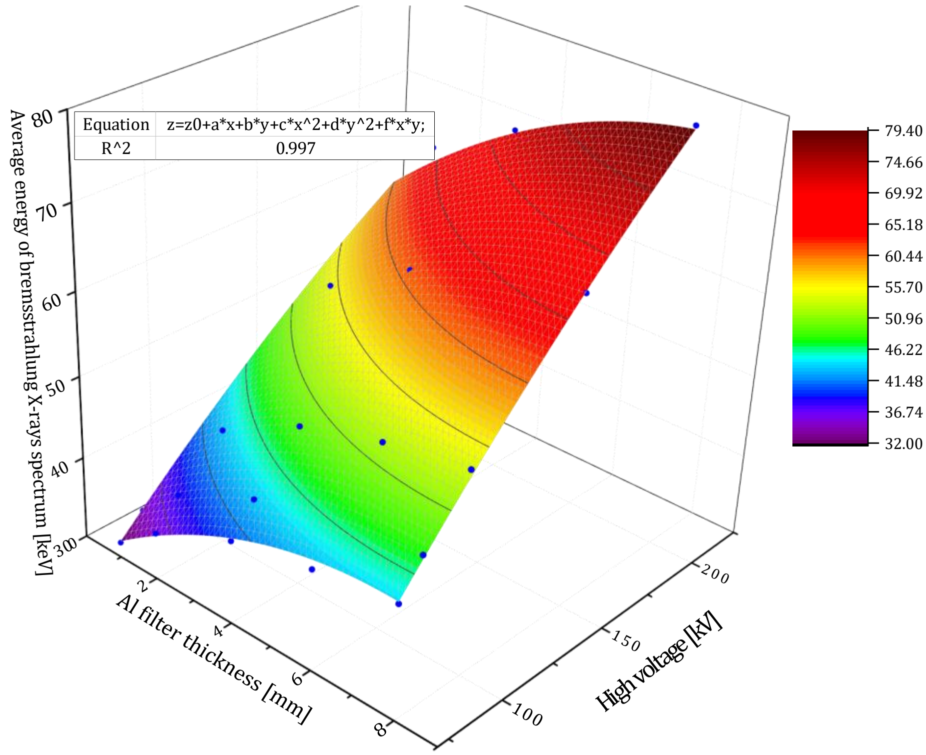

Figure 1. Plot of $\bar{E}_X$ as a function of Al filter thickness and high voltage

Table 2 shows that the percentage differences between the fitted and calculated values remain consistently low, mostly within ±3%, with many differences closer to ±1%. This small magnitude of deviation across all combinations of high voltage (70 to 200 kV) and Al filter thickness (1 to 8 mm) indicates a strong agreement between the fitted model and the calculated data. This close agreement clearly reflects the accuracy and dependability of the data fitting.

Table 2. Comparison of average energy of bremsstrahlung X-ray spectra between fitted and calculated values

High voltage, Te [kV]	Al filter thickness (mm)									
	1	Diff. [%]	2	Diff. [%]	4	Diff. [%]	6	Diff. [%]	8	Diff. [%]
70	32.0	0.5	35.4	-0.8	40.5	0.8	43.4	2.7	44.2	0.5
80	34.7	0.8	38.1	-2.2	43.4	-0.6	46.4	1.3	47.3	-1.1
100	39.7	1.4	43.3	-1.7	48.9	-1.1	52.2	-0.5	53.4	-1.8
150	51.0	1.9	54.9	-0.5	61.2	0.0	65.4	1.3	67.3	0.3
200	60.4	1.6	64.7	-2.6	71.7	-0.4	76.6	1.2	79.3	-0.5

Diff. is the percentage difference between fitted and calculated values

4. Conclusion

This study effectively characterized the average energy of the bremsstrahlung X-ray spectrum of the Rigaku-200EGM generator. The results indicated that increasing the aluminum filter thickness and tube voltage significantly raises the average energy of the bremsstrahlung X-rays. A mathematical model developed using a nonlinear surface fitting method accurately predicts the average energy of the bremsstrahlung spectrum for Al filter thicknesses from 1 to 8 mm and tube high voltages between 70 and 200 kV, achieving a high correlation coefficient (R^2 = 0.997) with minimal deviation from the calculated data. This result will be valuable for calculating the radiation dose of bremsstrahlung X-rays and for studying the half-value layer of the Rigaku-200EGM X-ray generator.

Declaration of Conflicting Interests

The authors declare no potential conflicts of interest with respect to the research, authorship and publication of this article.

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References

1. J. T. Bushberg, J. A. Seibert, E. M. Leidholdt, J. M. Boone, The essential physics of medical imaging, Lippincott Williams & Wilkins, 2012.
2. Quyet, P. D, Select a model to represent the bremsstrahlung X-ray spectrum for Rigaku-200EGM X-ray generator, International Journal of Science, Architecture, Technology, and Environment 02(09), (2025) 22-28.
3. F. M. Khan, J. P. Gibbons, The physics of radiation therapy, Lippincott Williams & Wilkins, 2014.
4. G. F. Knoll, Radiation Detection and Measurement, Wiley, 2010.
5. D. Q. Pham, Calculation of equivalent dose rate of bremsstrahlung X-rays generated by Rigaku-200EGM X-ray generator, International Journal of Scientific Research and Engineering Development 8(4), (2025) 2025-2029.
6. E. J. Hall, A. J. Giaccia, Radiobiology for the Radiologist, Lippincott Williams & Wilkins, 2012.
7. F. H. Attix, Introduction to radiological physics and radiation dosimetry, Wiley-VCH, 2008.
8. W. R. Hendee, E. R. Ritenour, Medical imaging physics, Wiley-Liss, 2002.