International Journal of
Science, Architecture, Technology and Environment

3-Total Sum Cordial Labeling for Open and Closed Diagonal Ladder Graph

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DOI: https://doi.org/10.63680/ijsate0625016.07

Abstract

In graph theory labeling is the assignment of labels to vertices and edges based on certain condition. For a graph $G = (V(G), E(G))$, In this paper deal with the vertex labeling function $f : V(G) \rightarrow \{0, 1, 2\}$ For each edge uv, assign the label $[f(u) + f(v)](mod3)$.Then the map f is called 3-Total sum cordial labeling of G if $|(v_f(i) + e_f(i)) - (v_f(j) + e_f(j))| \leq 1$; $i, j \in \{0, 1, 2\}$.Where $v_f(i)$ and $e_f(i)$ denotes the total number of vertices and edges with $i = \{0, 1, 2\}$.In this paper, We prove some graphs like Open Diagonal ladder graph $O(DL_n)$,Closed Diagonal ladder graph DL_n are 3-Total sum cordial graphs.

Keywords: Cordial labeling, Sum cordial labeling,3-Total sum cordial labeling, 3-Total sum cordial graphs

Subject Classification: 05C78

1. Introduction

The graphs considered here are finite, undirected and simple. For all standard terminology and notation follow Harrary [4]. Let $G(V, E)$ be a graph where the symbols $V(G)$ and $E(G)$ denote the vertex set and edge set respectively. Graph labeling serves as a frontier between number theory and application of graphs.

A dynamic survey of graph labeling is regularly updated by Gallian [2]. Cordial graph was introduced by Cahit[1] as a weaker version of both graceful and harmonious graphs. The concept of sum cordial labeling of graphs was introduced by Shiama J.[6]. The concept of 3-Total sum cordial labeling of graphs was introduced by Ghosh, Poulomi and Ghosh, Sumonta and Pal,Anita[3]. The concept of 3-Total edge sum cordial labeling of graphs was introduced by Tenguria Abha and Verma Rinku [7]. The edge product cordial labeling of graphs was introduced by S.K.Vaidya and Barasara C.M [8]. 3-total sum cordial labeling of graphs is a specific type of graph labeling that combines concepts from cordial labeling and total sum labeling. We give a brief summary of definitions which are useful for the present investigation.

2. Preliminaries

Definition 2.1

Let $G = (V, E)$ be a graph, and let $f : V \longrightarrow \{0,1\}$ be a labeling of its vertices, and let the induced edge labeling $f^* E \longrightarrow \{0,1\}$ be given by $f^*(uv) = (f(u) + f(v))(\text{mod}2)$, where $e = uv(\in E)$ and $u, v \in V$. Let v_0 and v_1 be the numbers of vertices that are labeled by 0 and 1, respectively, and let e_0 and e_1 be the corresponding numbers of edges. Such a labeling is called cordial if both $|v_0 - v_1| \le 1$ and $|e_0 - e_1| \le 1$ hold. A graph is called cordial if it admits cordial labeling.

Definition 2.2

Let G be a graph. Let f: V(G) → {1, 2, 3}. For each edge uv, assign the label [f(u) + f(v)] (mod3). Then the map f is called 3 – sum cordial labeling of G if | vf(i) – vf(j)| ≤ 1 and | ef(i) – ef(j)| ≤ 1; i, j ∈ {0, 1, 2}, where f(i) denotes the total number of vertices and edges with i = {0, 1, 2}.

Definition 2.3

A total cordial labeling of a graph G with vertex set and edge set as an cordial labeling such that number of vertices and edges labeled with 0 and the number of vertices and edges labeled with 1 differ by at most 1, i.e., $|(v_0 + e_0) - (e_1 + v_1)| \le 1$. A graph with a total cordial labeling is called a total cordial graph.

Definition 2.4

Let G be a graph. Let f : V (G) → {0, 1, 2} For each edge uv, assign the label [f (u) + f (v)](mod3).Then the map f is called 3-Total sum cordial labeling of G if |f (i) – f (j)| ≤ 1; i, j ∈ {0, 1, 2},where f (x) denotes the total number of vertices and edges with x = {0, 1, 2}.

Definition 2.5

A ladder L n is a graph with $V(L_n) = \{u_i , v_i \ / \ 1 \le i \le n\}$ and $E(L_n) = \{u_i u_{i+1}, v_i v_{i+1} \ / \ 1 \le \mathbf{i} \le n - 1\} \{ u_i , v_i \ / \ 1 \le \mathbf{i} \le n - 1\}$.

Definition 2.6

An open Diagonal ladder graph $O(DL_n)$, n≥ 2 is a Ladder graph with 2n vertices and is got from an open ladder graph OL_n with the additional edges $v_i u_i + 1 : 1 \le i \le n - 1$.

Definition 2.7

An closed Diagonal ladder graph DL_n, $n \geq 2$is a ladder graph with 2n vertices and is got from a closed ladder graph Ln with the additional edges $v_i u_i + 1$; $1 \leq i \leq n - 1$.

3. Main Result

Theorem 3.1. The Open Diagonal Ladder $O(DL_n)$ is 3-Total sum cordial graph , for $n \geq 2$.
Proof.

Let the vertex set of Open Diagonal ladder graph is $V(G) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$. To define $f : V(O(DL_n)) \rightarrow \{0, 1, 2\}$.We consider two cases as follow

Case 1: $n \equiv 0 (mod 3)$

Let $n = 3t, t \in N$

Define

$f(v_{3i+1}) = 1; 0 \leq i < t$
$f(v_{3i+2}) = 0 ; 0 \leq i < t$
$f(v_{3i+3}) = 2; 0 \leq i < t$

Define

$f(u_{3i+1}) = 1; 0 \leq i < t$
$f(u_{3i+2}) = 0; 0 \leq i < t$
$f(u_{3i+3}) = 2; 0 \leq i < t$

Case 2: $n \equiv 1 (mod 3)$
Let $n = 3t + 1, t \in N$

Define
$f(v_{3i+1}) = 0; 0 \leq i < t$
$f(v_{3i+2}) = 1; 0 \leq i < t$
$f(v_{3i+3}) = 2; 0 \leq i < t$

Define
$f(u_{3i+1}) = 1; 0 \leq i < t$
$f(u_{3i+2}) = 2; 0 \leq i < t$
$f(u_{3i+3}) = 0; 0 \leq i < t$

Case 3: $n \equiv 2 (mod 3)$
Let $n = 3t + 2, t \in N$

Define

$f(v_{3i+1}) = 2;\ 0 \le i < t$
$f(v_{3i+2}) = 1;\ 0 \le i < t$
$f(v_{3i+3}) = 0;\ 0 \le i < t$

Define

$f(u_{3i+1}) = 0;\ 0 \le i < t$
$f(u_{3i+2}) = 2;\ 0 \le i < t$
$f(u_{3i+3}) = 1;\ 0 \le i < t$

Thus in each cases, we have $|f(i) - f(j)| \le 1$, $i \ne j$; $i, j \in \{0, 1, 2\}$. Hence the Open Diagonal Ladder $O(DL_n)$ is a 3-Total sum cordial graph.

Example 3.2. The Open Diagonal Ladder $O(DL_5)$ is 3-Total sum cordial graph

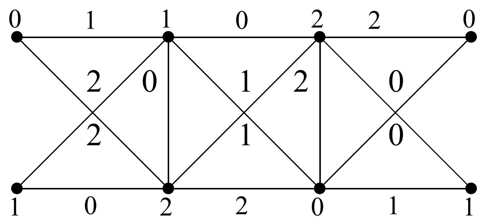

Figure 1: 3-Total sum cordial labeling of Open Diagonal Ladder $O(DL_n)$

3.3 Table

Vertex and Edge conditions for 3-Total sum cordial labeling of Open Diagonal Ladder $O(DL_n)$

Case	Edge condition	Vertex conditions	$f(i) = v_f(i) + e_f(i)$
$n = 3t$	$vf(0) = 2t$ $vf(1) = 2t$ $vf(2) = 2t$	$ef(0) = 5t - 1$ $ef(1) = 5t - 2$ $ef(2) = 5t - 2$	$f(0) = 7t - 1$ $f(1) = 7t - 2$ $f(2) = 7t - 2$
$n = 3t + 1$	$vf(0) = 2t + 1$ $vf(1) = 2t + 1$ $vf(2) = 2t$	$ef(0) = 5t$ $ef(1) = 5t - 1$ $ef(2) = 5t$	$f(0) = 7t + 1$ $f(1) = 7t - 1$ $f(2) = 7t$
$n = 3t + 2$	$vf(0) = 2t + 1$ $vf(1) = 2t + 1$ $vf(2) = 2t$	$ef(0) = 5t + 1$ $ef(1) = 5t + 1$ $ef(2) = 5t + 1$	$f(0) = 7t + 2$ $f(1) = 7t + 2$ $f(2) = 7t + 1$

Theorem 3.4. The Closed Diagonal Ladder (DL_n) is 3-Total sum cordial graph, for $n \ge 2$

Proof.
Let the vertex set of Closed Diagonal ladder graph is $V(G) = \{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\}$. To define $f : V(DL_n) \to \{0, 1, 2\}$. We consider two cases as follow.

Volume 02, Issue 07, July 2025

Case 1: $n \equiv 0 (mod3)$

Let $n = 3t, t \in N$

Define

f (v3i+1) = 2; 0 ⩽ i <t
f (v3i+2) =; 10 ⩽ i <t
f (v3i+3) = 0; 0 ⩽ i <t

Define

f (u3i+1) = 0; 0 ⩽ i < t
f (u3i+2) = 2; 0 ⩽ i < t
f (u3i+3) = 1; 0 ⩽ i <t

Case 2: $n \equiv 2(mod3)$

Let n = 3t + 2, $t \in$ N

Define

f (v3i+1) = 1; 0 ⩽ i <t
f (v3i+2) = 2; 0 ⩽ i <t
f (v3i+3) = 0; 0 ⩽ i <t

Define

f (u3i+1) = 1; 0 ⩽ i < t
f (u3i+2) = 2; 0 ⩽ i < t
f (u3i+3) = 0; 0 ⩽ i <t

Thus in each cases, we have |f (i) − f (j)| ≤ 1, i ≠ j; i, j ∈ {0, 1, 2}.
Hence the Closed Diagonal Ladder DLn is a 3-Total sum cordial graph.

Remark: The Closed Diagonal Ladder DLn is not a 3-Total sum cordial graph if n ≡ 1(mod3)

Example 3.5. The Closed Diagonal Ladder (DL5) is 3-Total sum cordial graph

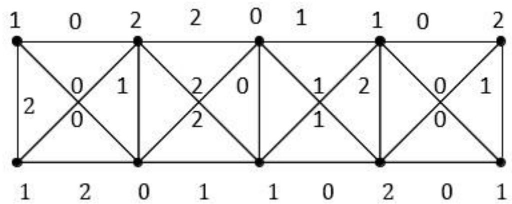

Figure 2: 3-Total sum cordial labeling of Closed Diagonal Ladder (DL_5)

3.6 Table

Vertex and Edge conditions for 3-Total sum cordial labeling of Closed Diagonal Ladder (DL_n)

Case	Edge condition	Vertex conditions	$f(i) = v_f(i) + e_f(i)$
$n = 3t$	$vf(0) = 2t$ $vf(1) = 2t$ $vf(2) = 2t$	$ef(0) = 5t - 2$ $ef(1) = 5t - 1$ $ef(2) = 5t - 1$	$f(0) = 76t - 2$ $f(1) = 7t - 1$ $f(2) = 7t - 1$
$n = 3t + 2$	$vf(0) = 2t$ $vf(1) = 2t + 2$ $vf(2) = 2t + 2$	$ef(0) = 5t + 4$ $ef(1) = 5t + 1$ $ef(2) = 5t + 1$	$f(0) = 7t + 4$ $f(1) = 7t + 3$ $f(2) = 7t + 3$

4. Importance and Applications

3-Total sum cordial labeling is a recent extension that explores new combinatorial properties of graphs. It contributes to the broader field of graph labeling by :

- Providing insight into balanced labeling schemes.
- Offering potential applications in symmetric network design
- Assisting in coding and cryptographic systems where balanced label assignments are desired.
- Supporting theoretical investigations I modular arithmetic and combinatorics.

5. Conclusion

3-Total sum cordial labeling enriches the field of graph labeling with its elegant combination of modular arithmetic and balanced condition. It offers a platform for both theoretical exploration and practical applications in areas requiring symmetry and balance. In this paper we have discussed about 3-Total sum cordial labeling for open and closed diagonal ladder graphs. We contribute some results to the theory of these 3-Total sum cordial labeling. Examples are provided at the end of each theorem for better understanding of the labeling pattern defined in each theorem.

Declaration of Conflicting Interests

The authors declare no potential conflicts of interest with respect to the research, authorship and publication of this article.

Funding

The author received no financial support for the research, authorship and publication of this article.

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