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## Stress Analysis of an Infinite Isotropic Wedge Under Antiplane Deformation

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### Abstract

A homogenous infinite isotropic elastic wedge under anti-plane deformation, is investigated in this paper, for fracture analysis. The infinite mellin transform is used to formulate the problem. A closed form solution for displacement, for each of the two cases of boundary conditions studied, is obtained. From these solutions, the geometric strength of stress singularities for some configurations of the material, and their corresponding stress intensity factors are analyzed. The stress intensity factor for the cases, when the Tada et al method is applicable is shown to be in agreement with the Shahani method (which is more in general). It was observed that similar configurations respond differently under different load conditions.

**Keywords:** Stress Analysis, Infinite Isotropic Wedge, Deformation, Stress Intensity, Displacement

### 1. Introduction

This study deals with the antiplane shear deformation fracture parameter response of a homogenous infinite isotropic wedge, with its apex at the origin. The material occupies the region expresses in cylindrical polar coordinate  $(r, \theta, z)$

$r \geq 0, 0 \leq \theta \leq \phi, -\infty < z < \infty$  (See Fig. 1)

Two problems were analyzed to show that the results given by some researchers, as corollaries to cases from finite wedges, can be derived directly from infinite wedges [1, 2]. In another study, it was shown that the stresses near the apex of an isotropic elastic wedge are proportional to  $r^{-\lambda}$ ,  $0 < \lambda < 1$  [3]. Many researchers have tried to find and interpret the influence of  $\lambda$  in fracture deformation [4-6]. Another study was conducted on

finite and infinite elastic wedges, focusing on the stress distribution and deformation behaviour. They found that the stress field near the apex of a wedge exhibits a singular behaviour, characterized by a power-law dependence on the distance from the apex. The study revealed that the stress concentration near the apex of the wedge is more pronounced for finite wedges than for infinite wedges [7]. Ma and Hou identified two primary deformation modes for elastic wedges, viz.: bending and torsion. The study showed that the deformation patterns of finite and infinite wedges differ significantly, with finite wedges exhibiting more complex deformation behaviour. The findings of Ma and Hou's study have important implications for the design of structural components, such as joints and connections, where elastic wedges are commonly encountered. The results have relevance to fracture mechanics, where the stress distribution near the apex of a wedge can play a critical role in determining the likelihood of crack propagation. In all, Ma and Hou's study provides valuable insights into the behaviour of elastic wedges, which can inform the design of structural components and the analysis of fracture mechanics problems.

A group of three scholars, applied Mellin transform techniques to analyze the stress distribution in a specific type of mechanical component [8]. They derived closed-form solutions for the Mellin transforms of certain functions, which can be used to solve related problems in mechanics and engineering. The study demonstrated the application of Mellin transform techniques to fracture mechanics problems, providing insights into the stress distribution near cracks and other defects (Shahani, 2007). Similarly, another devoted scientist developed and applied Mellin transform techniques to analyze and solve problems involving stress distribution, crack propagation and other related engineering topics [9]. The study provided analytical solutions for various problems using Mellin transforms, which can be used to validate numerical solutions, and provide insights into the underlying physics. Shahni's work demonstrated the potential of Mellin transforms in solving real-world engineering problems, particularly in the fields of solid mechanics and material science. Shahani's study on Mellin transforms appears to be a research paper focused on the application of Mellin transforms to solve problems in mechanics and engineering.

A trio in their scholarly work, provide comprehensive solutions for stress intensity factors ( $K$ ), for various crack geometries and loading conditions. They presented detailed analyses of crack tip stress fields, including the effects of plasticity and non-linear material behaviour [10]. Their study covers the fundamental principles of fracture mechanics, including the concepts of stress intensity factor, fracture toughness, and crack growth. It is a seminal work in the field of fracture mechanics. Nnadi (2009) investigated the properties of Mellin transforms, including their relationship to other integral transforms, such as the Laplace and Fourier transforms. The study demonstrated the use of Mellin transforms to solve differential equations, particularly those with variable coefficients [11]. Nnadi's work showed the potential of Mellin transforms in solving real-world engineering problems, such as those involving heat transfer, vibration, and stress analysis. The work used infinite Mellin transform to study inhomogeneity in an elastic cylinder. Real  $\lambda$  has been used to define geometric strength of stress singularity and to formulate the stress intensity factor relation different from the early one given by Tada, Paris & Irwin (1985).

Two researchers conducted a study on low temperature strength deformation failure of turbine shaft produced with rhus fibre reinforced polyester polymer composite. Failure analysis on turbine shaft produced with rhus reinforced polyester polymer composite was successfully carried out [12]. The turbine blades are subjected to maximum tangential stress of  $2.5 \text{ N/mm}^2$ , at locations very close to the root of the blades. The displacement suffered at the tip of each blade on account of 'attack' from nozzle jet, have limited value of 0.4 mm. Deformation simulation results reveal that, dynamically, both runner and shaft assembly of the turbine would survive design loadings under actual service condition. An academic synergy by some scholars

produced a work on micro-mechanics mercerization analysis on the tensile strength and interphase quality of stipa stem fibre-reinforced polypropylene composite materials [13]. The composites were tensile tested and the impact of the proposed treatments was evaluated. Later, the Kelly-Tyson modified equation and a modified rule of mixtures – the micro-mechanic models – were used to study the impact of such treatments on the quality of the interphase between the polymer and the reinforcement. Both treatments showed a high impact on the tensile strength and the quality of the interphase. Further research by three academics investigated fluid structure analysis of a micro-hydrokinetic turbine rotor blade. Simulations were carried out using a commercial CFD solver. One-way fluid structure interaction technique was implemented to solve the quasi-static field problem [14]. The study examined two model scale blades with different material properties (structural steel and e-glass epoxy blade). The mechanical properties of the modal scale turbine blades are little and could be considered reasonably negligible. The steel blade material performed better due to its high stiffness property. Results show that the predicted maximum von-Mises stress for the two scale models is less than their tensile yield strength property. Fracture mechanics analysis assumes that the pull-out process involves the propagation of a crack or fracture along the interface between the reinforce material and matrix material. This was what Madu & Okoronkwo (2018) sought to investigate in their work areca fiber-reinforced polyester bio-composites pull-out analysis. TGA measurements showed that with the NaOH treatment, the fiber becomes more thermally resistant [15].

## 2. General Equations

From the constitutive equations for this type of deformation, the non-vanishing stresses are the polar stresses, denoted by:

$$\sigma_{\theta z}(r, \theta) = \frac{\mu}{r} \frac{\partial w}{\partial \theta}(r, \theta) \text{ -----} \quad (1)$$

$$\sigma_{rz}(r, \theta) = \mu \frac{\partial w}{\partial r}(r, \theta) \text{ -----} \quad (2)$$

$W(r, \theta)$  is the only non-vanishing component of displacement in the z-direction and  $\mu$  is the material shear modulus. The equation of equilibrium produces Laplace equation in terms of  $w(r, \theta)$ , given by;

$$\frac{\partial^2 w}{\partial r^2}(r, \theta) + \frac{1}{r} \frac{\partial w}{\partial r}(r, \theta) + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2}(r, \theta) = 0 \quad (r \geq 0, 0 \leq \theta \leq \phi) \text{ -----} \quad (3)$$

By application to (3) of the infinite Mellin transform of  $w(r, \theta)$  defined by

$$\bar{w}(S, \theta) = \int_0^\infty r^{s-1} w(r, \theta) dr \text{ -----} \quad (4)$$

We get;

$$\frac{d^2 \bar{w}}{d\theta^2}(S, \theta) + S^2 \bar{w}(S, \theta) = 0 \text{ -----} \quad (5)$$

Provided that;

$$\lim_{\substack{n \rightarrow \infty \\ r \rightarrow 0}} [r^{s+1} \frac{\partial w}{\partial r}(r, \theta) + r^s \bar{w}(S, \theta)] = 0 \text{ -----} \quad (6)$$

The asymptotic behaviours are, expressed by another researchers, thus [16];

$$\sigma_{rz}(r, \theta), \sigma_{\theta z}(r, \theta) = O(r^{-\lambda}), 0 < \lambda < 1 \quad r \rightarrow 0$$

$$O(r^{-1-\epsilon}), \epsilon > 0 \quad r \rightarrow \infty \quad \text{-----} \quad (7)$$

Once (4) is derived, the displacement sought for is obtained, through the inversion formula given by;

$$w(r, \theta) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \bar{w}(S, \theta) r^{-s} ds, \lambda - 1 < C < \epsilon \quad \text{-----} \quad (8)$$

The bound on C, is obtained by use of (6) and (7)

### 3. The First Problem

The geometry of the first problem is shown in Figure 1, below.

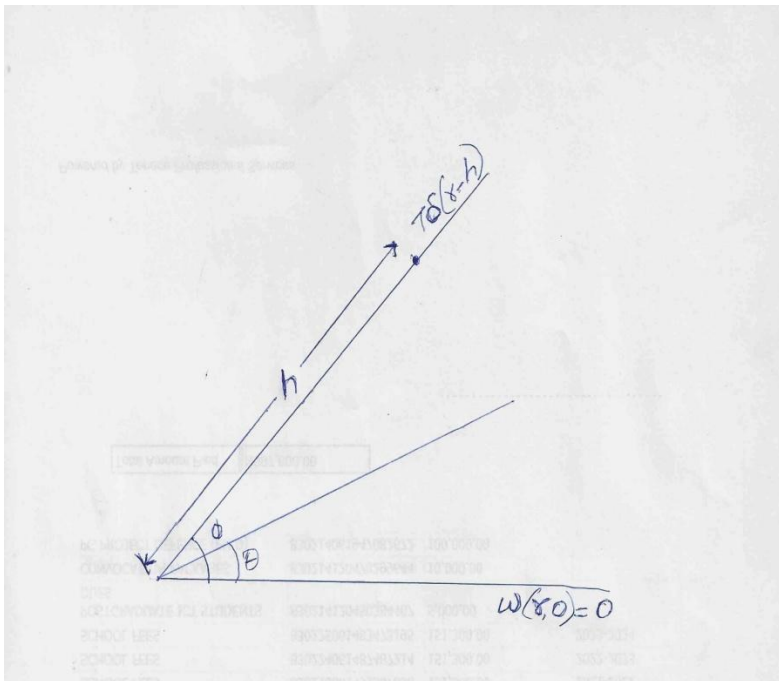


**Figure 1:** Geometry of the First Problem

The first problem studies the parameters using the traction-displacement boundary condition, given as;

$$\sigma_{\theta z}(r, \theta) = T\delta(r-h), \quad 0 \leq \theta \leq \pi \quad \text{-----} \quad (9)$$

$$w(r, 0) = 0, \quad r \geq 0 \quad \text{-----} \quad (10)$$

where  $\delta$  is the Dirac delta function

The S-plane problem to be solved is given by (5), subject to the boundary conditions

$$\frac{d\bar{w}}{d\theta}(S, \phi) = \frac{Th^s}{\mu} \quad (11)$$

$$\bar{W}(S, 0) = 0 \quad (12)$$

where (11) and (12) are the Mellin transform of (9) and (10)

Let us assume the solution of (5) in the form

$$\bar{W}(S, \theta) = A(s) \cos \theta s + B(s) \sin \theta s \quad (13)$$

Then,

$$\frac{d\bar{W}}{d\theta}(S, \theta) = -sA \sin \theta s + sB(s) \cos \theta s \quad (14)$$

Application of (11) and (12) to (13) and (14), produces

$$A(s) = 0, \text{ and}$$

$$B(s) = \frac{Th^s}{\mu s \cos \phi s}$$

Therefore, (13) becomes

$$\bar{w}(S, \theta) = \frac{Th^s \sin \theta s}{\mu s \cos \phi s} \quad (15)$$

Substituting (15) into (8) yields

$$w(S, \theta) = \frac{T}{\mu} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\sin \theta s}{\cos \phi s} \left(\frac{r}{h}\right)^{-s} ds, \quad -\frac{1}{2} < C < \frac{1}{2} \quad (16)$$

The strip of analyticity has been considered with  $\lambda = \epsilon = \frac{1}{2}$ . The integral in (16) can be evaluated by residue theory and Jordan's contour closure Lemma.

The integral, d, has simple poles located at;

$$S_{tn} \simeq \pm \left(\frac{2n+1}{2\phi}\right) \pi, n=0, 1, 2, 3, \dots$$

We shall be interested in fracture responses at wedge apex ( $r \rightarrow 0$ ). Therefore, the stress distribution in the

region  $r \leq h$  will be analyzed. The relevant poles are simple and located at;

$$S_n \simeq -\left(\frac{2n+1}{2\phi}\right)\pi, n=0, 1, 2, \dots$$

The residues are;

$$\frac{2}{\pi} \frac{(-1)^n}{2n+1} \sin(2n+1) \frac{\pi\theta}{2\phi} \left(\frac{r}{h}\right)^{(2n+1)\pi/2\phi} \dots\dots\dots (17)$$

The corresponding displacement is given by;

$$w(r, \theta) = \frac{2T}{\mu\pi} \sum_{n=0}^{\infty} \frac{-1^n}{2n+1} \sin \frac{(2n+1)}{2\phi} \pi\theta \left(\frac{r}{h}\right)^{\left(\frac{2n+1}{2\phi}\right)\pi}, r \leq h \dots\dots\dots (18)$$

Use of (1), yields

$$\sigma_{\theta z}(r, \theta) = \frac{T}{h\phi} \sum_{n=0}^{\infty} (-1)^n \cos \frac{(2n+1)}{2\phi} \pi\theta \left(\frac{r}{h}\right)^{\left(\frac{2n+1}{2\phi}\right)\pi-1}, r \leq h \dots\dots\dots (19)$$

From (2)

$$\sigma_{\theta z}(r, z) = \frac{T}{h\phi} \sum_{n=0}^{\infty} (-1)^n \sin \frac{(2n+1)}{2\phi} \pi\theta \left(\frac{r}{h}\right)^{\left(\frac{2n+1}{2\phi}\right)\pi-1}, r \leq h \dots\dots\dots (20)$$

### 3.1 Geometry and Load Singularities at the Apex

Equations (19) and (20) indicate dependence of stress distribution on the apex angle showing that, for this mode of loading, the stresses respond differently at the apex according to the geometric and load singularities involved. The stress due to geometry is analyzed by use of the dominant term in (19) and (20) to study the factor  $r^{\frac{(2n+1)}{2\phi}n-1}$  as  $r \rightarrow 0$  for stress boundedness within specific ranges of  $\phi$ .

When  $n=0$ , the dominant term occurs and the series becomes

$$\sigma_{\theta z}(r, \theta) = \frac{T}{\phi} h^{\frac{\pi}{2d}} \cos \left(\frac{\pi\theta}{2\phi}\right) r^{\frac{\pi}{2\phi}-1}, r \rightarrow 0 \dots\dots\dots (21)$$

$$\sigma_{rz}(r, \theta) = \frac{T}{\phi} h^{\frac{\pi}{2d}} \sin \left(\frac{\pi\theta}{2\phi}\right) r^{\frac{\pi}{2\phi}-1}, r \rightarrow 0 \dots\dots\dots (22)$$

The load singularities are associated with the stress intensity factor,  $K_{111}$ , which can be derived from the relations postulated by Tada, Paris & Irwin (1985).

$$\begin{aligned} \sigma_{\theta z}(r, \theta) &= \frac{K_{111}}{\sqrt{2\pi r}} \sin \frac{\theta}{2}, r \rightarrow 0 \\ \sigma_{rz}(r, \theta) &= \frac{K_{111}}{\sqrt{2\pi r}} \cos \frac{\theta}{2}, r \rightarrow 0 \dots\dots\dots (23) \end{aligned}$$

Another stress intensity factor formula that incorporates geometric singularity, is the one by Shahani (2007), given as;

$$K_{111} = \lim_{n \rightarrow 0} \sqrt{2\pi} r^{\lambda_s} \sigma_{\theta z}(r, \phi - \pi) \text{-----} \quad (24)$$

where  $\lambda_s$  is the geometric strength of stress singularity at the wedge apex defined by

$$\lambda_s = 1 - s_o$$

where  $s_o$  is the least simple zero of  $\cos \phi_s$  when the stress is unbounded.

For this problem,

$$s_o = \frac{\pi}{2\phi}$$

We take cognizance of the following assumptions:

i) If  $0 < \phi \leq \frac{\pi}{2}$ , the figure is part of, or equal to the first quarter of the plane. For  $0 < \phi < \frac{\pi}{2}$ ,

$$\frac{\pi(2n+1)-1}{2\phi} > 2n, \text{ is satisfied.}$$

When  $n=0$ , we have;

$$\frac{\pi}{2\phi} - 1 > 0$$

Therefore, by (21) and (22), the stresses are bounded at the apex (as  $r \rightarrow 0$ ), for figures determined by  $0 < \phi < \frac{\pi}{2}$  under the prescribed load conditions.

When  $\phi = \frac{\pi}{2}$ , we get  $\lambda_s = 0$

The material is a quarter plane with the entire positive x-axis fixed and shear concentrated load applied along the positive y-axis. The apex is the junction of the axes  $x \geq 0$  and  $y \geq 0$ , which is shown not to be singular.

ii) If  $\frac{\pi}{2} < \phi \leq \pi$ , the geometric figure is the first quadrant to which part of, or the entire second quadrant is added.

When  $\frac{\pi}{2} < \phi < \pi$ , the inequality  $\frac{\pi(2n+1)-1}{2\phi} < 2n$ , is satisfied

The dominant term when  $n=0$ , gives  $\frac{\pi}{2\phi} - 1 < 0$ .

Hence (21) and (22) indicate that the stresses are unbounded at the apex.

The case of  $\phi = \pi$ , implies that  $\lambda_s = \frac{1}{2}$ .

The configuration is the upper half plane, with the positive x-axis fixed, and shear concentrated load applied at a distance  $h$  units from the origin. The effect of the load singularity is assessable by methods of Tada, Paris & Irwin (23), due to the form of the stresses and Shahani (24). Equation (21) or (22) by Tada method, produces;

$$K_{111} = T \left( \frac{2}{\pi h} \right)^{\frac{1}{2}}$$

Utilizing (21) and (25) lead to;

$$K_{111} = \lim_{n \rightarrow 0} \sqrt{2\pi} r^{\lambda s} \sigma_{\theta z}(r, \phi) = T \left( \frac{2}{\pi h} \right)^{\frac{1}{2}}$$

This demonstrates equality of the two methods.

iii) If  $\pi < \phi \leq \frac{3\pi}{2}$ , the figure is the first, second and part of (or including) the third quadrant. When  $\pi < \phi \leq \frac{3\pi}{2}$ , the inequality  $\frac{\pi}{2\phi} (2n+1) - 1 < 2n$ , is satisfied. Considering  $n=0$ , when the dominant term occurs, we have  $\frac{\pi}{2\phi} - 1 < 0$ . Equations (21) and (22) indicate that the stresses are unbounded at the apex for this range. If  $\phi \leq \frac{3\pi}{2}$ , the material is the entire space without the fourth quadrant. The apex is the inter-section of axes  $x \geq 0$  and  $y \leq 0$  with the positive x-axis fixed and concentrated load applied at a distance  $h$  units from the origin on the negative y-axis.

For this case,  $\lambda s = \frac{2}{3}$

The stress intensity factor is calculated by use of the Shahani relation (24), and equation (21) to get;

$$K_{111} = \lim_{n \rightarrow 0} \sqrt{2\pi} r^{\frac{2}{3}} \sigma_{\theta z} \left( r, \frac{\pi}{2} \right) = T \left( \frac{2}{3\pi} \right)^{h^{-1/3}}$$

We cannot use the Tada equations because when  $\phi = \frac{3\pi}{2}$  is substituted into (21) or (22), none of the equations in (23) is obtained

iv) If  $\frac{3\pi}{2} < \phi < 2\pi$ , the configuration is the space without part of the fourth quadrant. The inequality  $\frac{\pi}{2\phi} (2n+1) - 1 < 2n$ , is satisfied. When  $n=0$ , use is made of (21) and (22), with  $\frac{\pi}{2\phi} - 1 < 0$  to see that the stresses are unbounded at the apex ( $r \rightarrow 0$ ). When  $\phi = 2\pi$ , the configuration resembles the space that has semi-infinite line crack along the positive real axis.

In this case,  $\lambda s = \frac{3}{4}$

The stress intensity factor is derived by use of (24) as;

$$K_{111} = \lim_{n \rightarrow 0} \sqrt{2\pi} r^{\frac{3}{4}} \sigma_{\theta z}(r, \pi) = \frac{T}{2} \pi^{1/2} h^{-\frac{1}{4}}$$

Which is the same as the one obtained as a special case of a finite wedge by Shahani (2007)

#### 4. The Second Problem

The second problem is the study of the parameters using the traction-traction boundary conditions given by;

$$\sigma_{\theta z}(r, \phi) = T \delta(r-h) \text{ ----- (25)}$$

$$\sigma_{\theta z}(r, \phi) = T \delta(r-h) \text{ ----- (26)}$$

The transform plane problem to be solved is (5), subject to the conditions

$$\frac{d\bar{w}}{d\theta}(s, \phi) = \frac{Th^s}{\mu} \text{ ----- (27)}$$

$$\frac{d\bar{w}}{d\theta}(s, \phi) = \frac{Th^s}{\mu} \text{ ----- (28)}$$

The geometry of the second problem is shown in Figure 2, below.

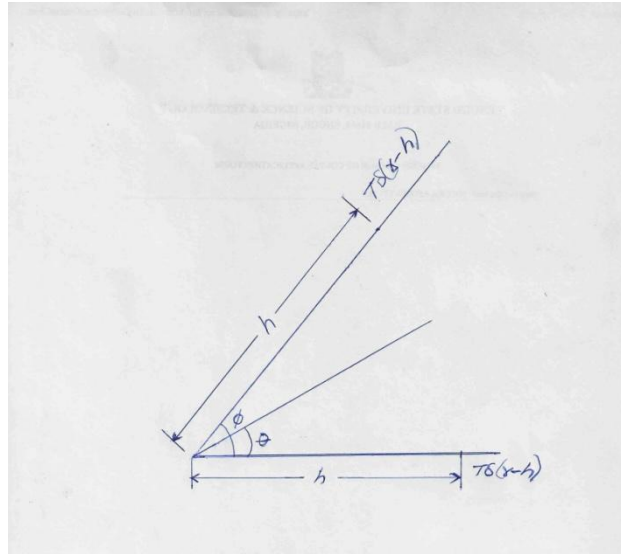


**Figure 2:** Geometry of the Second Problem

Where (27) and (28) are the Mellin transform of (25) and (26) respectively.

Assuming solution of the form (13) and deriving (14), then the application of (27) and (28) to them gives;

$$B(s) = \frac{Th^s}{\mu}, \text{ and } A(s) = \frac{Th^s}{\mu s \sin \phi s} [\cos \phi s - 1]$$

From (13), we get;

$$\bar{w}(s, \theta) = \frac{Th^s}{\mu s \sin \phi s} [\cos \phi s - 1] \cos \phi s + \frac{Th^s}{\mu s} [\sin \phi s] \text{ ----- (29)}$$

Because  $\frac{\sin \theta}{s}$  is an analytic function, Cauchy theorem gives;

$$w(r, \theta) = \frac{T}{\mu} \left[ \frac{1}{2\pi i} \right] \int_{c-i\infty}^{c+i\infty} \frac{(\cos \phi s - 1)}{s \sin \phi s} \cos \theta s \left( \frac{r}{h} \right)^{-s} ds$$

Residue method can be used to evaluate the integral. We will study the results in the region  $r \leq h$ , therefore, the simple poles of the integral that will be considered are those of the form;

$$S_n = \frac{-n\pi}{\phi}$$

The residue theorem gives the result as;

$$w(r, \theta) = \frac{2T}{\mu\pi} \sum_{n=0}^{\infty} \left( \frac{-1}{2n+1} \right)^{2n+1} \cos (2n+1) \frac{\pi\theta}{\phi} \left( \frac{r}{h} \right)^{\frac{(2n+1)\pi}{\phi}}, \quad r \leq h \text{ ----- (30)}$$

Hence,

$$\sigma_{\theta z}(r, \theta) = \frac{2T}{\phi h} \sum_{n=0}^{\infty} \sin (2n+1) \frac{\pi\theta}{\phi} \left( \frac{r}{h} \right)^{(2n+1)\frac{\pi}{\phi}-1}, \quad r \leq h \text{ ----- (31)}$$

$$\sigma_{rz}(r, \theta) = \frac{2T}{\phi h} \sum_{n=0}^{\infty} \cos (2n+1) \frac{\pi\theta}{\phi} \left( \frac{r}{h} \right)^{(2n+1)\frac{\pi}{\phi}-1}, \quad r \leq h \text{ ----- (32)}$$

#### 4.1 Geometry and Load Singularities at the Apex

As  $r \rightarrow 0$ , the dominant term occurs when  $n=0$ , then the stresses become

$$\sigma_{\theta z}(r, \theta) = \frac{2T}{\phi} h^{\frac{-\pi}{\phi}} \sin \frac{\pi\theta}{\phi} r^{\frac{\pi}{\phi}-1}, \quad r \rightarrow 0 \quad (33)$$

$$\sigma_{rz}(r, \theta) = \frac{2T}{\phi} h^{\frac{-\pi}{\phi}} \cos \frac{\pi\theta}{\phi} r^{\frac{\pi}{\phi}-1}, \quad r \rightarrow 0 \quad (34)$$

For this case,

$$\lambda s = 1 - \frac{\pi}{\phi}$$

We take cognizance of the following assumptions:

v.) If  $0 < \phi \leq \frac{\pi}{2}$ , the configuration is the same as in (assumption i) above. If  $0 < \phi < \frac{\pi}{2}$ , then  $\frac{\pi}{\phi} > 1$  or  $\frac{\pi}{\phi} - 1 > 0$ , the quarter plane is attained. If  $\phi = \frac{\pi}{2}$ , in which case  $\frac{\pi}{\phi} - 1 > 0$ , there is an indication that the stresses are bounded.

vi.) If  $\frac{\pi}{2} < \phi \leq \pi$ , the figure is the same as the one in (assumption ii) above.

If  $\frac{\pi}{2} < \phi < \pi$ , we have  $\frac{\pi}{\phi} > 1$  or  $\frac{\pi}{\phi} - 1 > 0$ . Hence, the stresses are bounded.

If  $\phi = \pi$ , the figure is the upper half plane with  $\theta=0$  not loaded, and  $\theta=\pi$  subjected to antiplane concentrated load at a distance  $h$  units from the origin. The apex is the origin which is not singular.

vii) If  $\pi < \phi < \frac{3\pi}{2}$ , the figure is the same as in (assumption iii) above.

For  $\pi < \phi < \frac{3\pi}{2}$ , the material is unbounded because;  $\frac{\pi}{\phi} - 1 < 0$

When  $\phi = \frac{3\pi}{2}$ , we have  $\frac{\pi}{\phi} = \frac{2}{3}$

Then,  $\lambda s = \frac{1}{3}$ , and  $\sigma_{\theta z}(r, \theta) = \frac{4T}{3\pi} h^{\frac{-2}{3}} \cos \frac{2}{3} \theta r^{\frac{-1}{3}}$

Which is not in the standard form for application of the Tada formula (23). For stress intensity factor, use is, then, made of the Shahani relations ((24) and (33), to get;

$$K_{111} = \lim_{n \rightarrow 0} \sqrt{2\pi} r^{\frac{1}{3}} \sigma_{\theta z} \left( r, \frac{\pi}{2} \right) = 2 \left( \frac{2}{3\pi} \right)^{1/2} h^{-\frac{2}{3}} T$$

viii) If  $\frac{3\pi}{2} < \phi \leq 2\pi$ , the configuration is the same as in (assumption ii) above.

For  $\frac{3\pi}{2} < \phi < 2\pi$ , we have  $\frac{\pi}{\phi} - 1 < 0$ , indicating that the stresses are unbounded, when  $\phi = 2\pi$ , the material looks like a semi-infinite radical crack along  $\theta=0$ .

In this case,  $\lambda s = \frac{1}{2}$ ,  $\frac{\pi}{\phi} = \frac{1}{2}$ ,

$$\sigma_{\theta z}(r, \theta) = \frac{2T}{2\pi} h^{\frac{-1}{2}} \sin \frac{\theta}{2} r^{\frac{-1}{2}}$$

Which is in the form to which (23) can be applied to get  $K_{111}$ . The result is;

$$K_{111} = \left( \frac{2}{\pi h} \right)^{1/2} T$$

The Shahani formula (24) gives

$$K_{111} = \lim_{n \rightarrow 0} \sqrt{2\pi} r^{\frac{1}{3}} \sigma_{\theta z} (r, \pi) = \left( \frac{2}{\pi h} \right)^{1/2} h^{-\frac{2}{3}} T$$

Which is the same result obtained by the Tada and by Shanani methods (as a special case of a finite wedge.

## 5. Conclusion

The stress analysis of an infinite elastic wedge under antiplane shear deformation has been made directly from the infinite material using the traction-displacement and traction-traction boundary condition. The stress distribution is shown to depend linearly on the wedge angle, and the applied traction. Different configurations with geometric discontinuities were identified and analyzed for geometric strength of stress singularity and corresponding stress intensity factor. It was shown that the results which Shahani (2007) obtained for an infinite wedge (as a special case of a finite body), is obtainable by direct use of an infinite wedge. Similar configurations respond differently under different loading. It was observed that where it is possible to derive the Tada et al. equation for stress intensity factor, the result was the same as the one obtained from the Shahani formula. However, the Shahani formula is applicable where the Tada et al. equation is not.

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